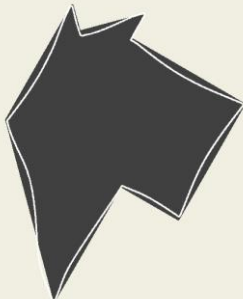
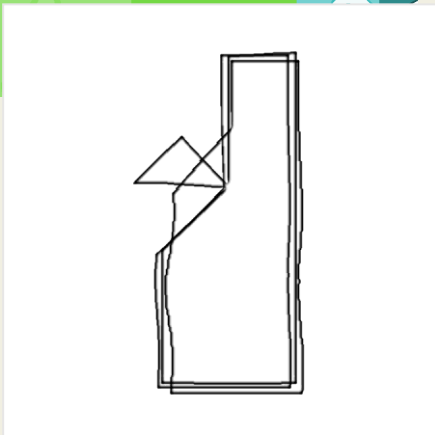
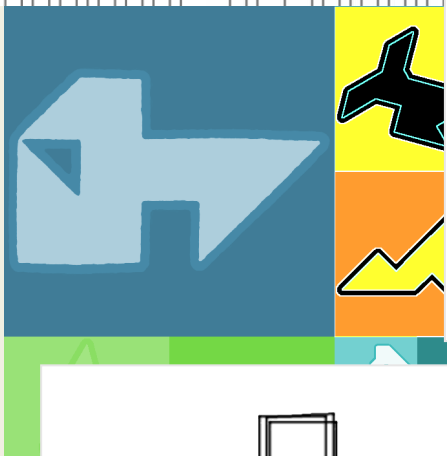
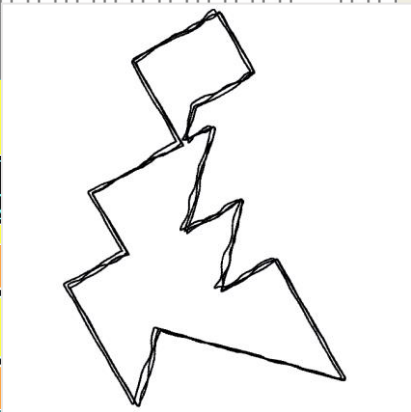
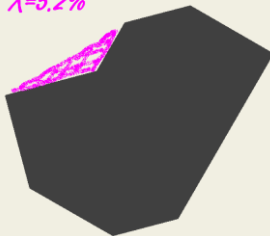


# WANDERINGS AROUND TANGRAM

Franco Cocchini



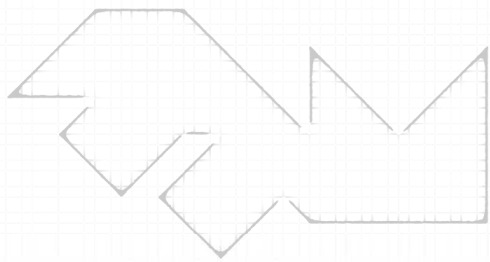
$X=5.2\%$



rotate by  $45^\circ$

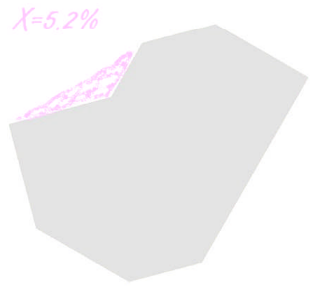
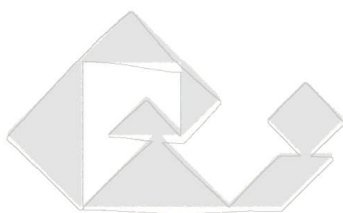
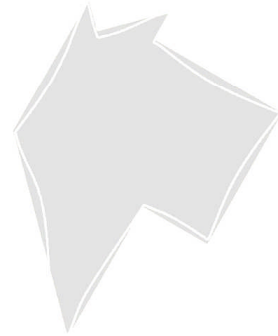
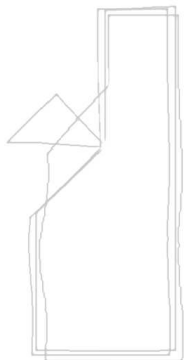
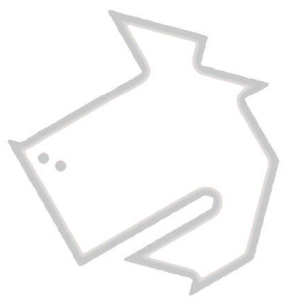
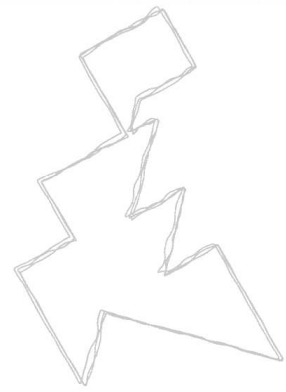
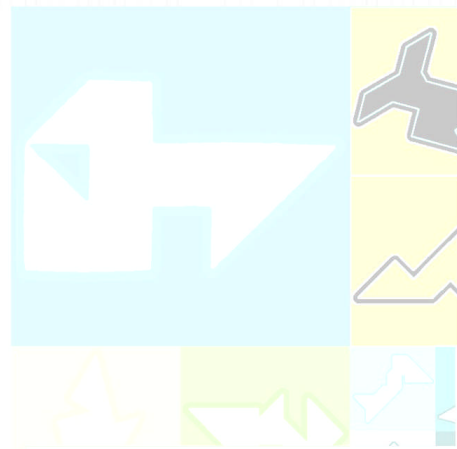
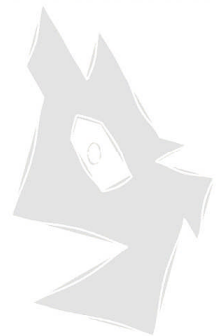






# WANDERINGS AROUND TANGRAM

Franco Cocchini



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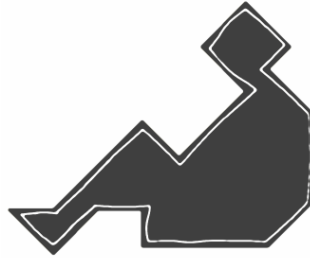
[www.lulu.com](http://www.lulu.com)

All the Tangram patterns have been obtained in digital form using the program Tanzzle v.1.0  
(Copyright © 2006 Franco Cocchini , TXu1-335-426)

[www.tanzzle.com](http://www.tanzzle.com)

Rheograms executable programs  
(Copyright © 2009-2010 Franco Cocchini)

[www.artistrising.com](http://www.artistrising.com)



*But it is another pastime altogether to create new and original designs of a pictorial character, and it is surprising what extraordinary scope the Tangrams afford for producing pictures of real life—angular and often grotesque, it is true, but full of character. I give an example of a recumbent figure that is particularly graceful, and only needs some slight reduction of its angularities to produce an entirely satisfactory outline.*  
Henry Ernest Dudeney, *Amusements in Mathematics*

## FOREWORDS

Tangram is a world apart, a fascinating world which can be appreciated from many points of view. It's a puzzle. It's a companion to learn geometry. It's a source of pictures. It's a source of art. It's a paradox. It's a metaphor. In any case, it's a way to activate the mind. This book deals with some sides of this polyhedron. It is an invitation to start (for beginners) or to continue (for experts) to enjoy Tangram. It is a report of my own exploration.

Tangram silhouettes often look like animals, humans, objects or symbols. Besides such resemblances, they have their own visual strength which has been of interest in Art, too. Donald Baechler, Patrick Scott and Matthew Langley are among the artists which used Tangram in some of their works. I pursued that way and I acknowledge their influence on my own work. I digitally reworked the images of this book to emphasize the visual impact, still preserving the game side.

Franco Cocchini

August 2010  
Salerno, Italy

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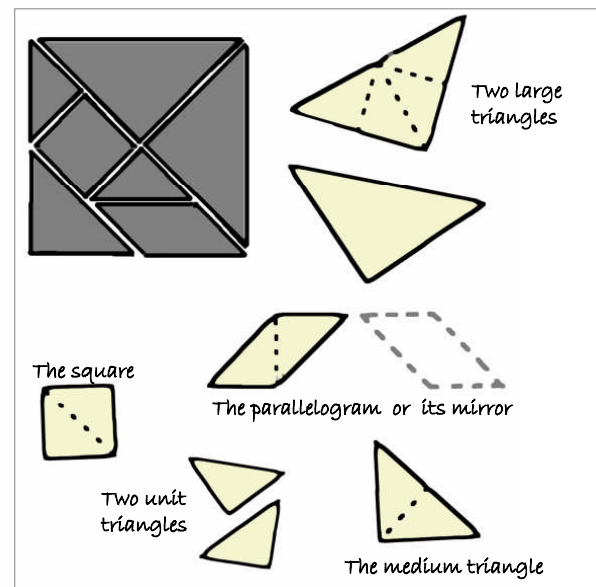
# 1 INTRODUCTION

Tangram is a puzzle, which consists of seven polygon tiles, sometimes called 'tan': a square, two small triangles, one medium triangle, two large triangles and a parallelogram.

The pieces can be obtained by cutting a large square. They can be rearranged on a plane in an infinite number of configurations, but all of them should be used and none should overlap the others.

A given pattern is usually displayed as a silhouette, in a way that matched sides cannot be distinguished. The player is challenged to reproduce that silhouette.

It's important to realize the relative proportions among the pieces: their relative sizes are fixed, not their absolute ones. The small triangles are the smaller units; the square, the medium triangle and the parallelogram can be obtained by joining two units. The two large triangles are obtained by joining four units. Therefore, the areas of the pieces are scaled as 1:2:4. I recommend to practice with a physical Tangram set, which you can purchase or make yourself.



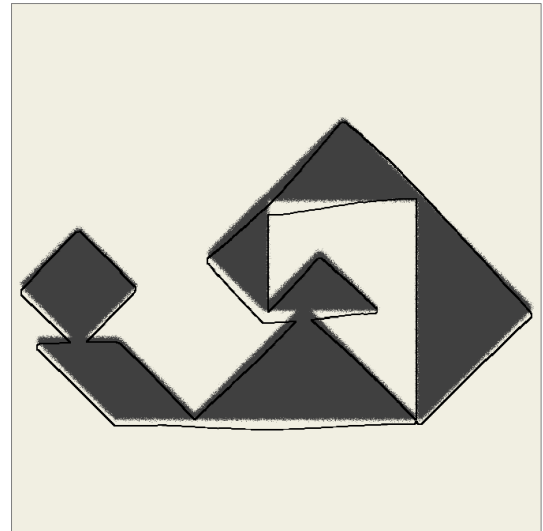
▲ Figure 1  
The seven Tangram pieces

*Many design problems can be posed with these games in mind; the main principle to be learned is that of economy of means - making the most of the least. Further, the game helps to sharpen the powers of observation through the discovery of resemblances between geometric and natural forms. It helps the student to abstract - to see a triangle, for example, as a face, a tree, an eye, or a nose, depending on the context in which the pieces are arranged. Such observation is essential in the study of visual symbols*

Paul Rand, *A Designer's Art*

## 2 Like a Tangram world

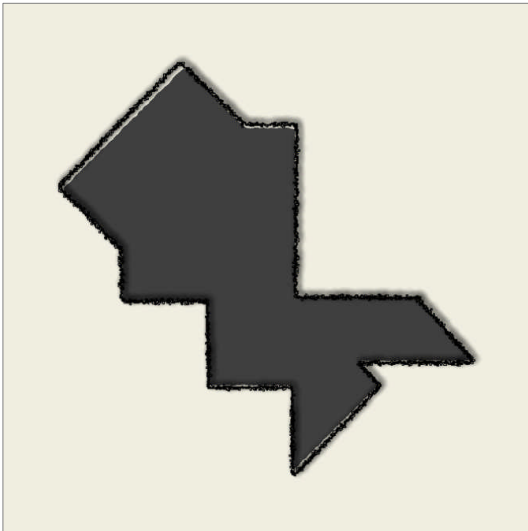
Tangram silhouettes often look like something: animals, humans, objects, and so on. As brain teasers, these kinds of figures are usually outspread shapes which mainly need a sense of proportion to be solved. They are often fascinating from a visual point of view, and they probably remain the best way to approach Tangram.



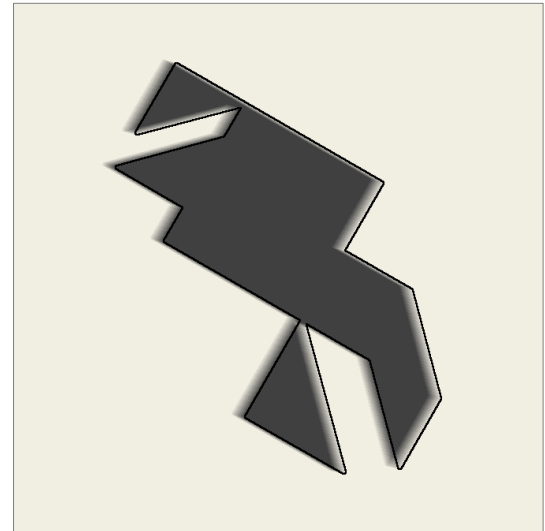
▲ 1

The snail. This is a quite simple silhouette; all but the pieces are distinguished and there is no need for a separate figure with the solution. It's an invitation to enjoy Tangram at your leisure.

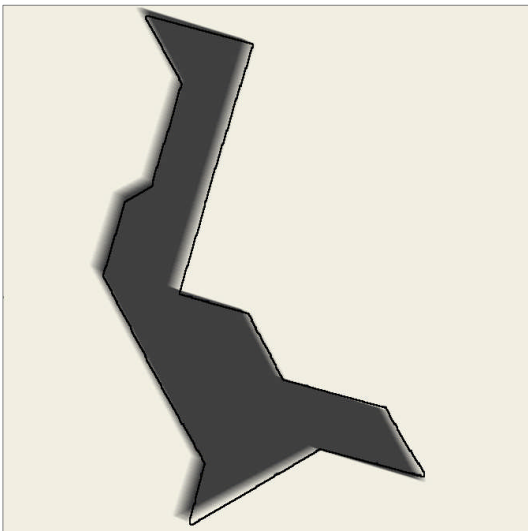




◀2  
Jumping frog #1



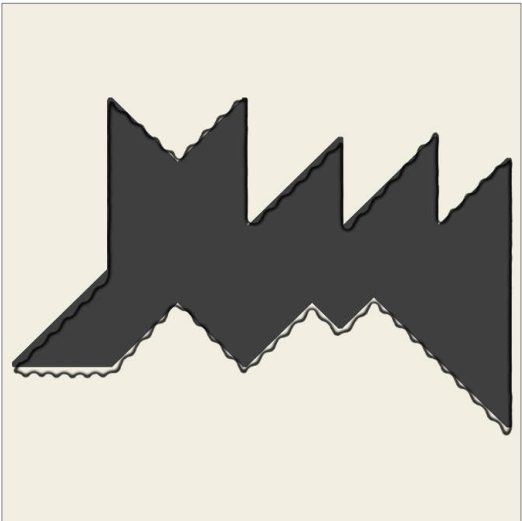
3▶  
Jumping frog #2



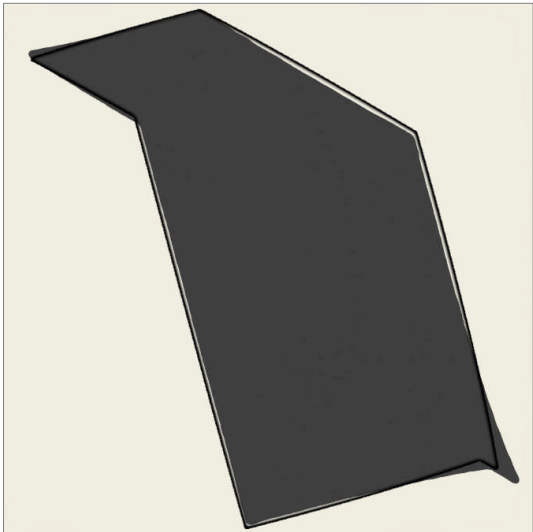
◀4  
Running ostrich



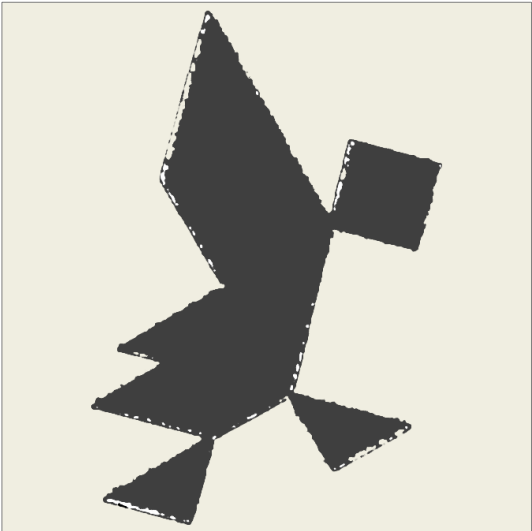
5▶  
Is it true that an  
ostrich, when  
pursued, hides its  
head in the sand  
and believes itself to  
be unseen?



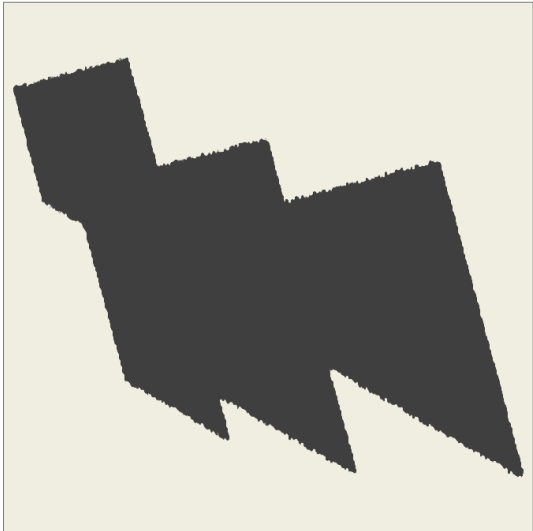
◀6  
Purring cat



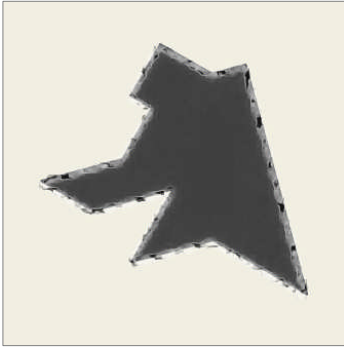
7▶  
Penguin



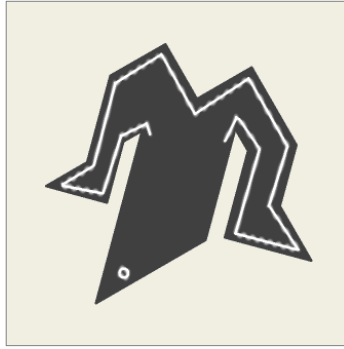
◀8  
Duck taking flight  
by Barry&Deb



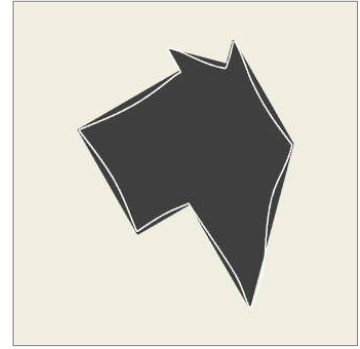
9▶  
Jellyfish



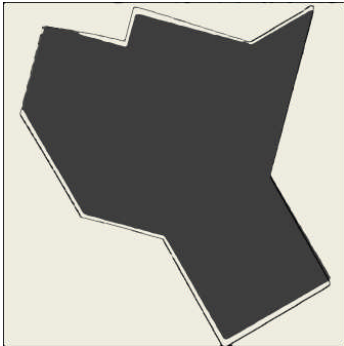
▲10 Pig



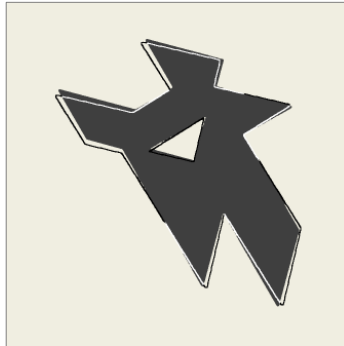
▲11 Mouflon



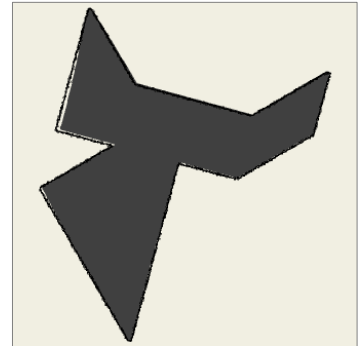
▲12 Dog #1



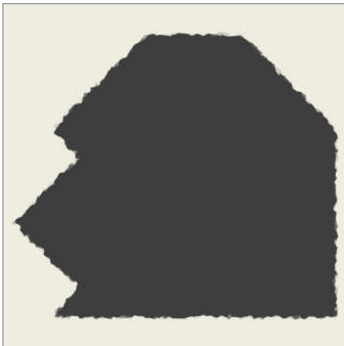
▲13 Dog #2



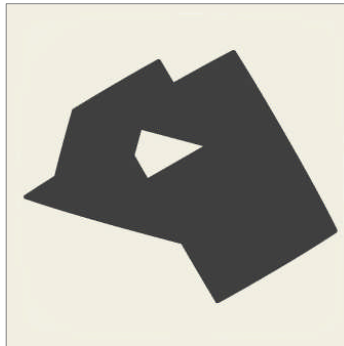
▲14 Goat



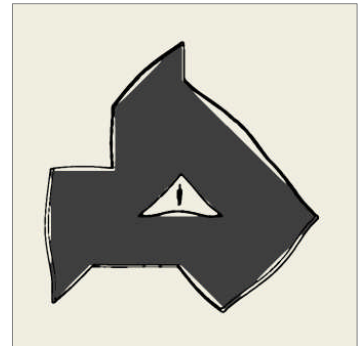
▲15 Gnu



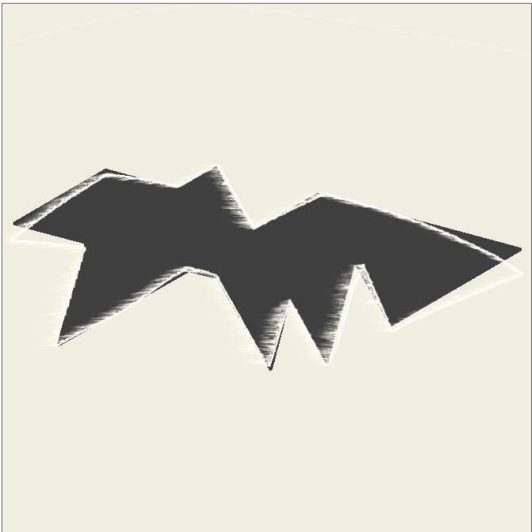
▲16 Gorilla



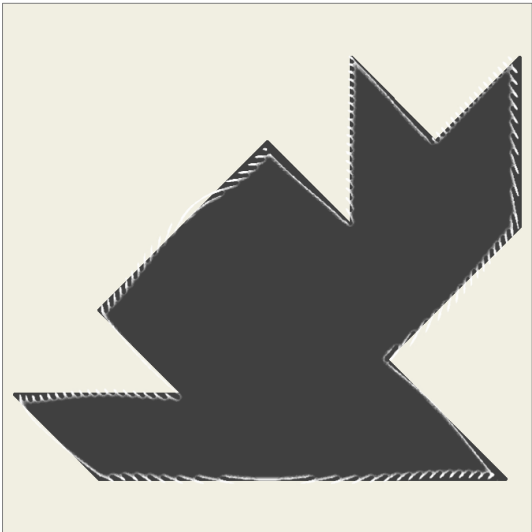
▲17 Pigeon



▲18 Bird of prey



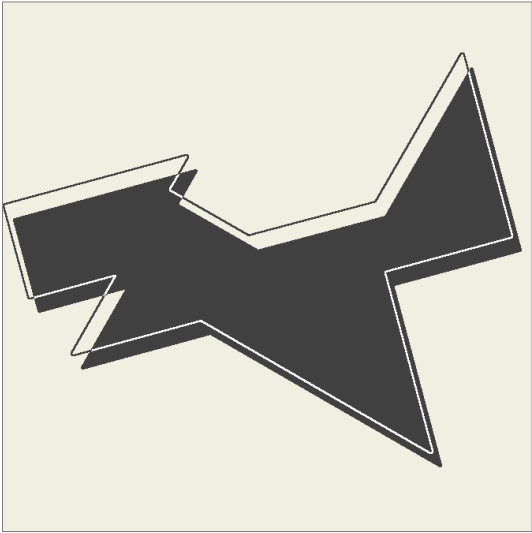
◀19  
Wild horse



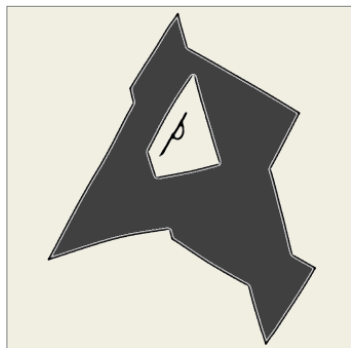
20▶  
Cat



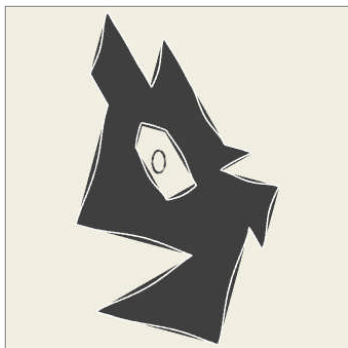
◀21  
Lobster



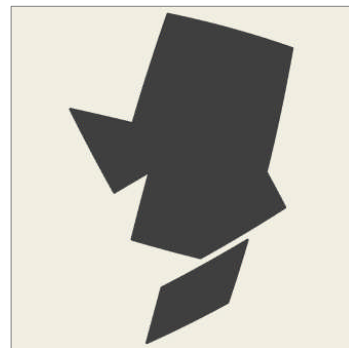
22▶  
Squirrel



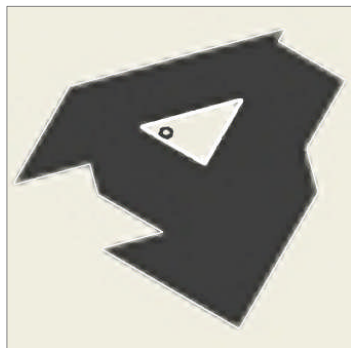
▲23



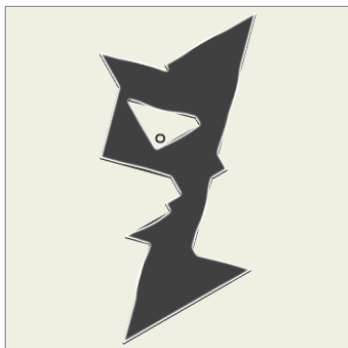
▲24



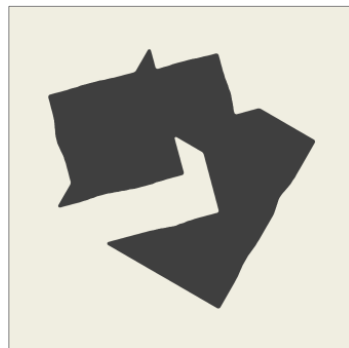
▲25



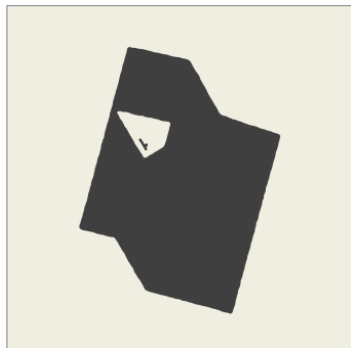
▲26



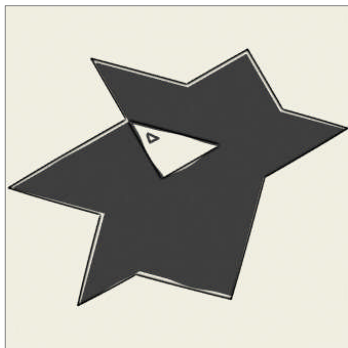
▲27



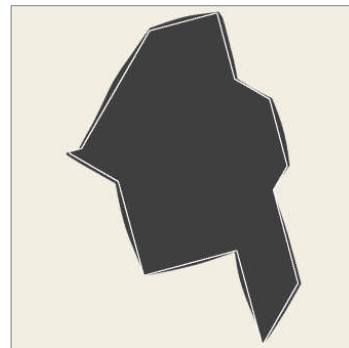
▲28



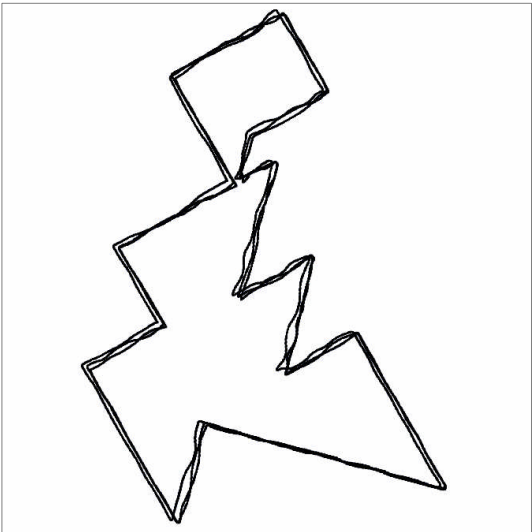
▲29



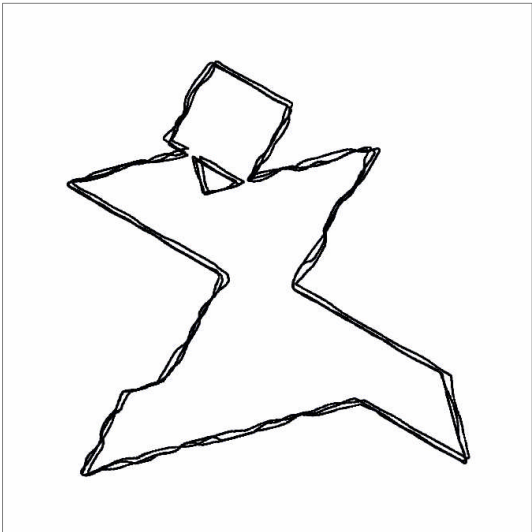
▲30



▲31



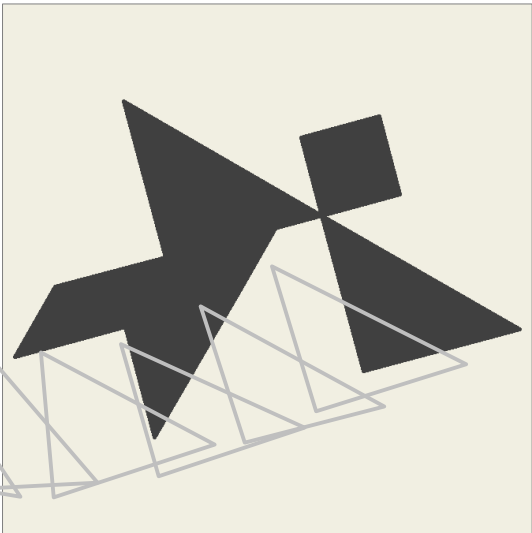
◀32  
Runnnig #1



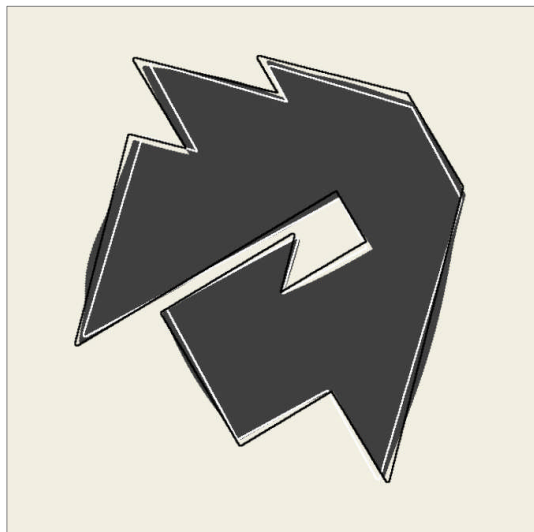
33▶  
Running #2



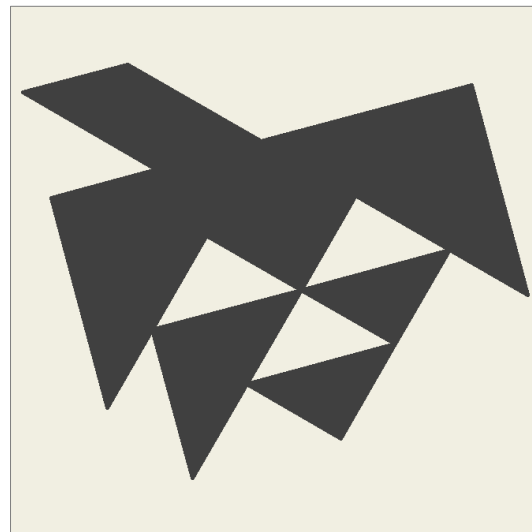
◀34  
Bowling #1



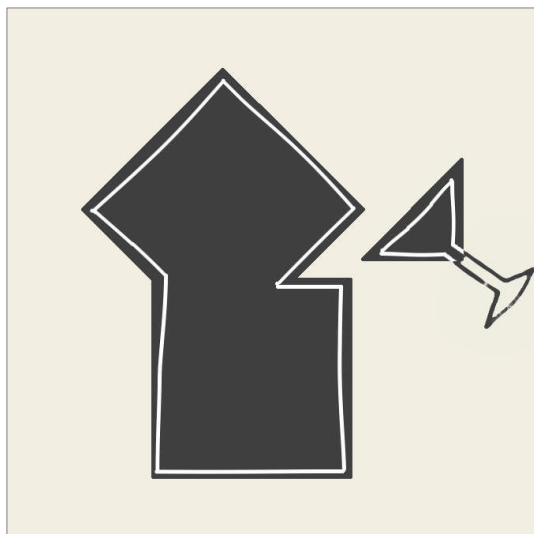
35▶  
Bowling #2



◀36  
Tufted head



37▶  
Mask

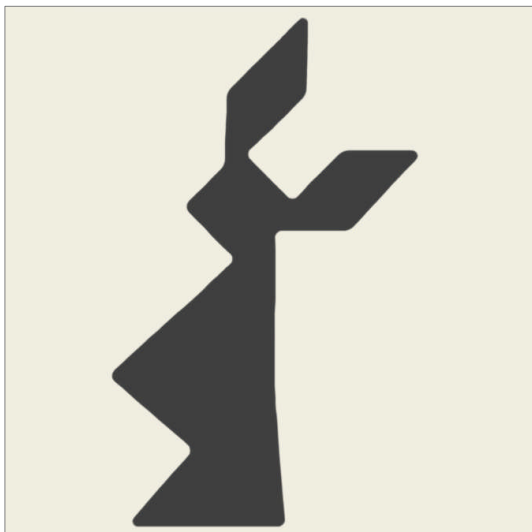


◀38  
Drinking



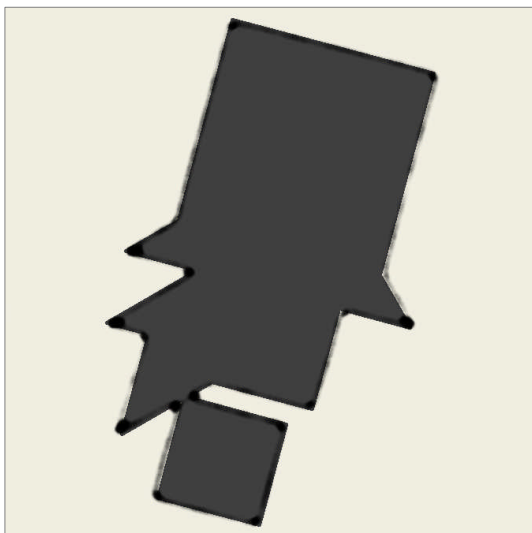
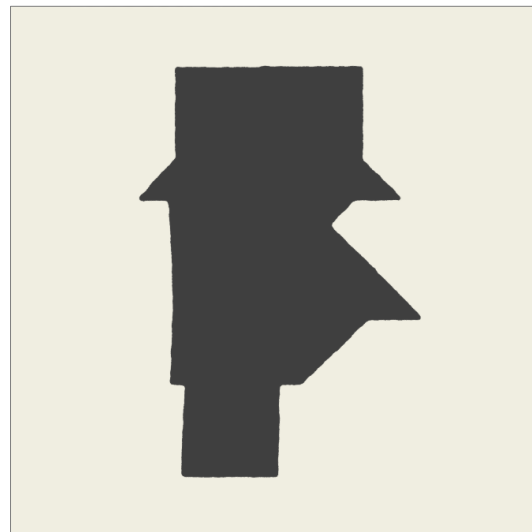
39▶  
Cross-legged seated man

## Chapter 2



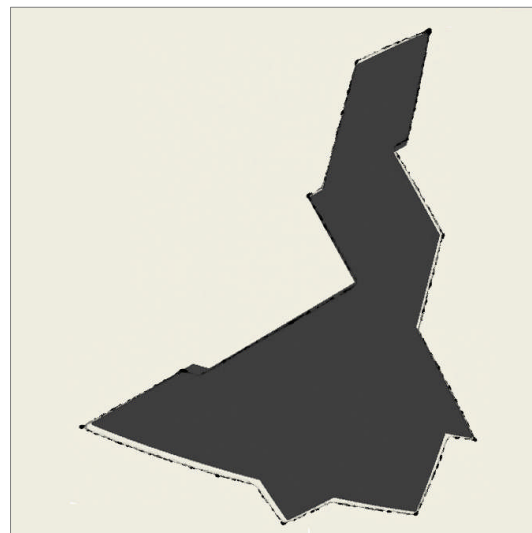
◀40  
The March Hare

41▶  
The Hatter  
From "Amusements in  
Mathematics" by  
Henry E. Dudeney  
"..As I have referred to  
the author of *Alice in  
Wonderland*, I give  
also my designs of  
March Hare and the  
Hatter."

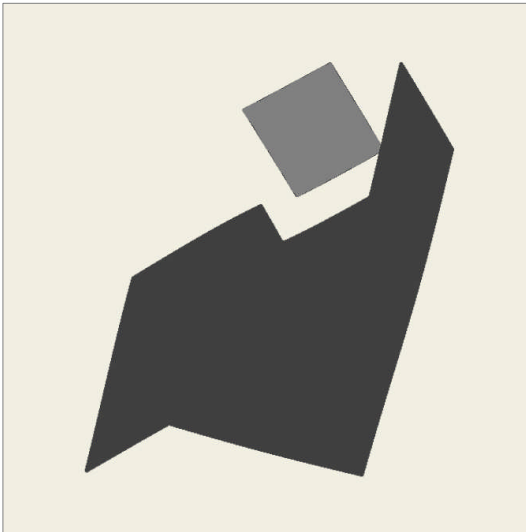


◀42  
The Hatter, as  
attributed to Lewis  
Carroll himself

43▶  
The Queen of Hearts  
"The Queen had only  
one way of settling all  
difficulties, great or  
small. 'Off with his  
head!' she said,  
without even looking  
round."

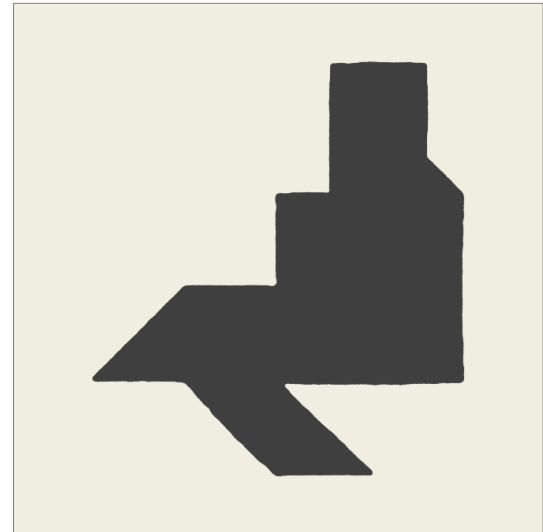






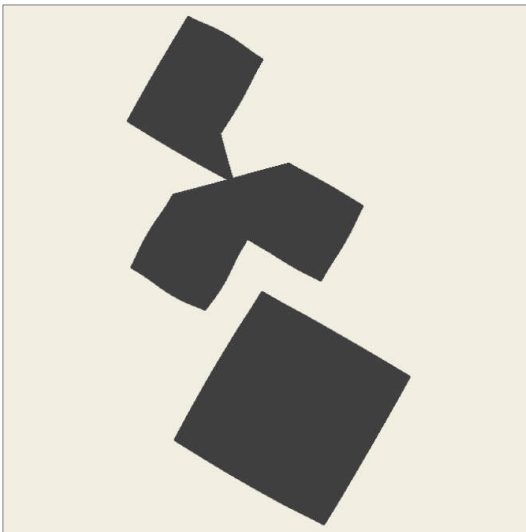
◀44

*Arrangement in  
Grey and Black:  
The Artist's Mother*  
by James McNeill  
Whistler, Musée  
d'Orsay Paris



45▶

*Tahitian Woman*  
by Paul Gauguin,  
Musée d'Orsay  
Paris

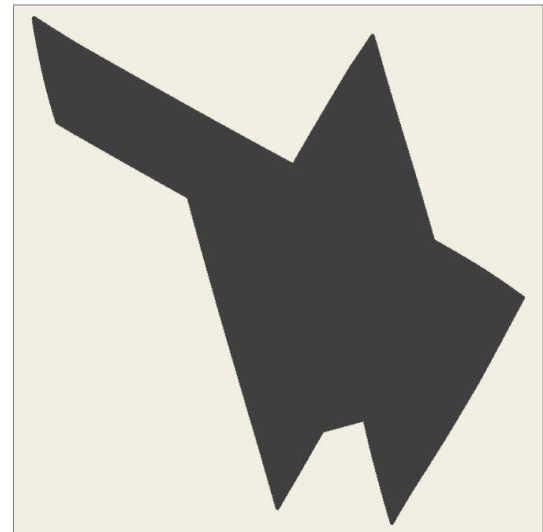


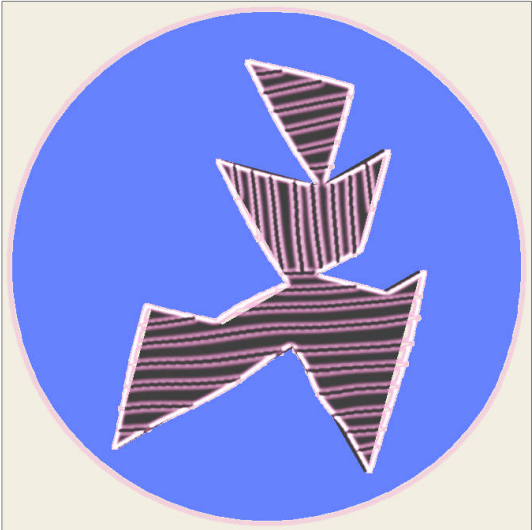
◀46

*Jessica Rabbit in  
Who Framed Roger  
Rabbit?* directed by  
Robert Zemeckis

47▶

*Horse and rider*  
after Marino Marini





◀48  
*Harlequin #1*,  
ceramic,  
17th century,  
Montelupo (Italy)



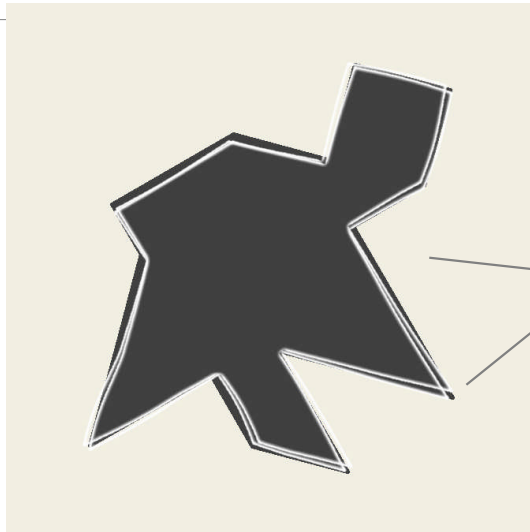
49▶  
*Harlequin #2*,  
ceramic,  
17th century,  
Montelupo (Italy)



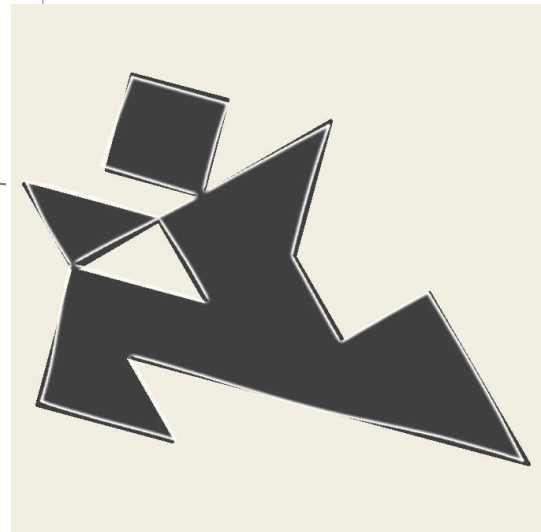
◀50  
Goya's *Maya*

51▶  
Hopper's *Nighthawk*

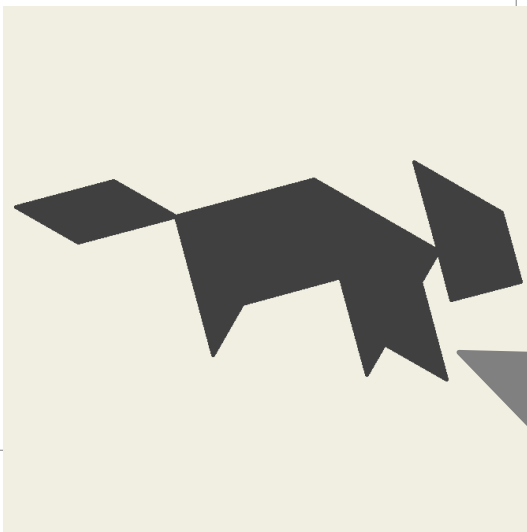




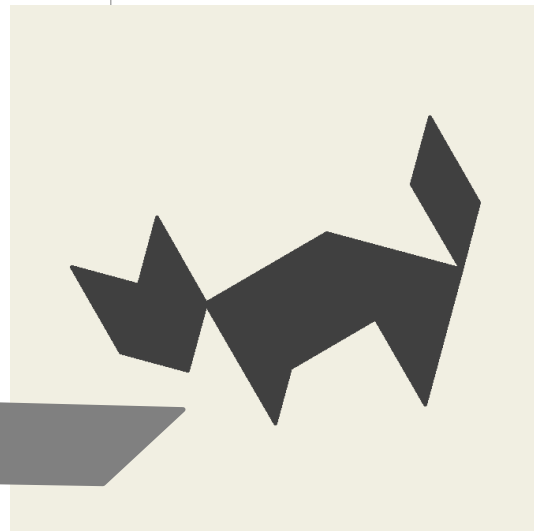
◀52  
Fencer #1

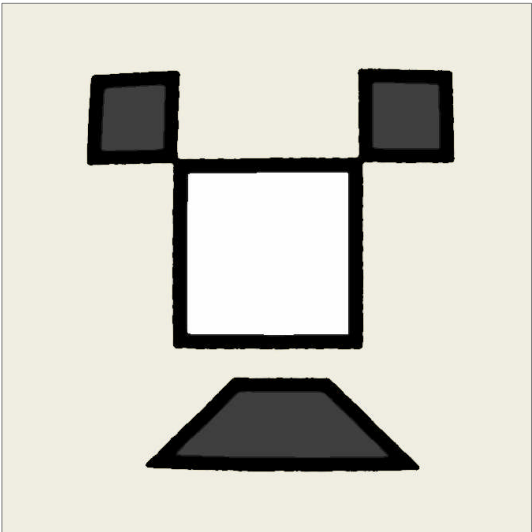


53▶  
Fencer #2

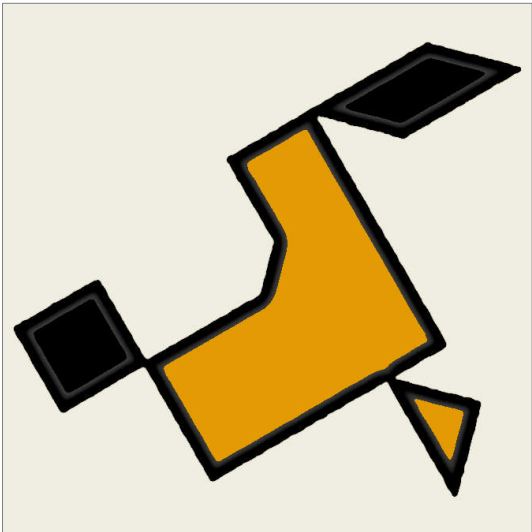


◀54  
55▶  
Kittens  
around a  
milk bowl,  
from "The  
eighth book  
of Tan" by  
Sam Loyd

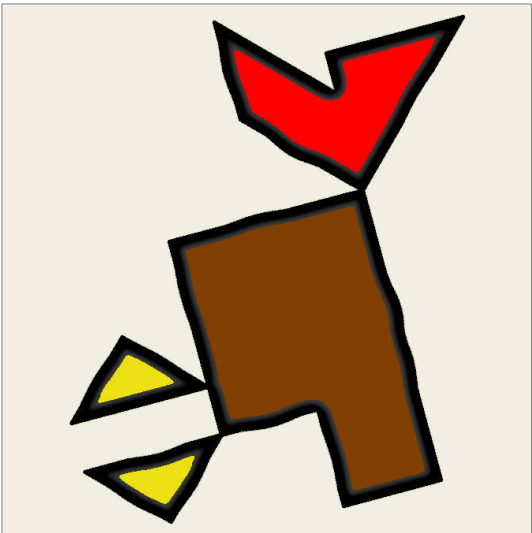




◀56  
Mickey Mouse

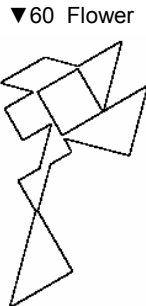


57▶  
Disney's Pluto

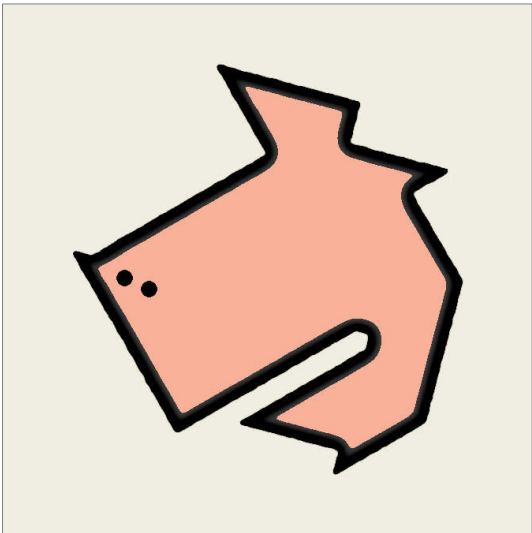


◀58  
Looney Tunes'  
Foghorn Leghorn

59▶  
Cartoon pig



▼60 Flower



### 3 Some Geometry and featurelessness

Tangram can be approached from a geometrical point of view. The Tangram pieces are polygons.

They are not bricks but still they are quite modular, their side lengths being proportional to 1,  $\sqrt{2}$ , 2 and  $2\sqrt{2}$ . That Golden sequence is probably the core of Tangram charm.

It is easy to identify dangling pieces (e.g. all the pieces in the snail silhouette N. 1). On the contrary, it's difficult to distinguish pieces which have one edge and one or two vertices matched on those of another piece. Still more puzzling is when two or more matched pieces exactly fit on the large edge of another piece.

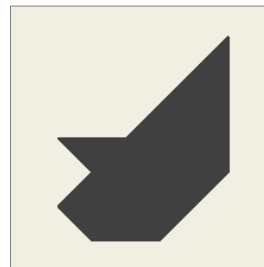
As a rule of thumb, hard-to-solve patterns are often built by matching as much edges and vertices of the pieces as possible, in order to have the silhouettes as featureless as possible. From this point of view, the convex figures should be the best.

A figure is called convex when every point on a line joining any two points of the figure lies within the figure itself. There exist 13 convex figure which can be obtained with the Tangram pieces (F. T. Wang and C. C. Hsung in the "American Mathematical Monthly", volume 19, 1942). No less, no more.

They have been collected in the fancy figure at page 16, with the companion solutions at page 55. For the puzzle players, at page 17, the convex silhouettes have been again displayed but as precise as they should, not reworked and distorted.

But, there are 16 figures!

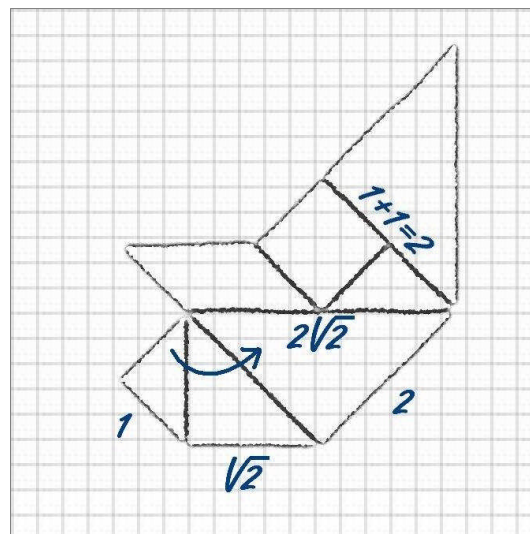
Before reading the followings, find the intruders.



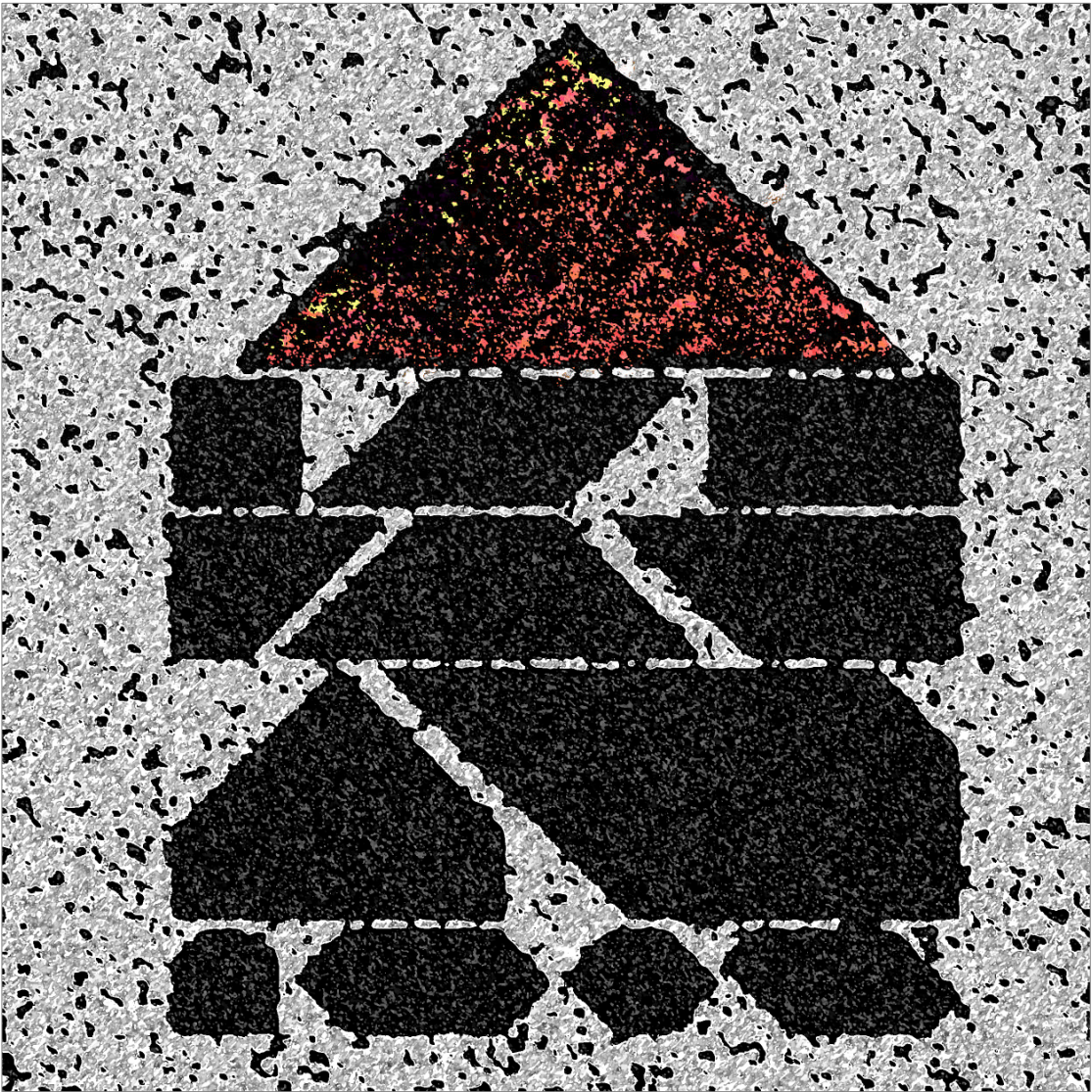
▲ 61

Example of fully matched pieces

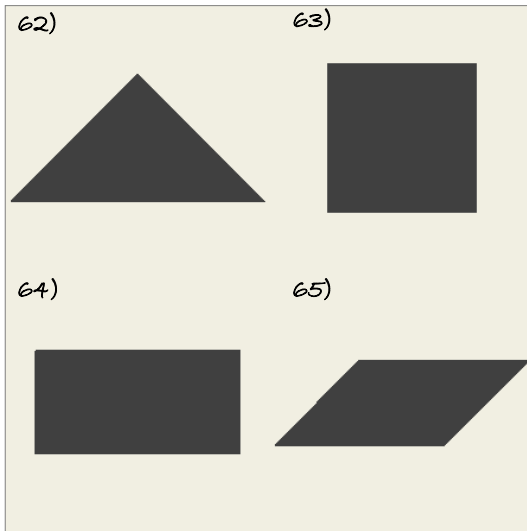
▼ Solution to silhouette N. 61





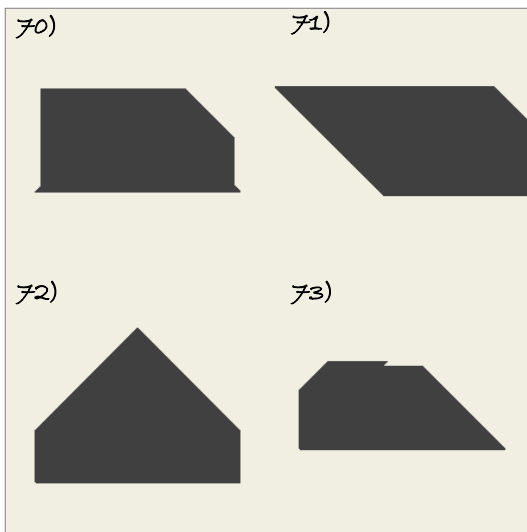
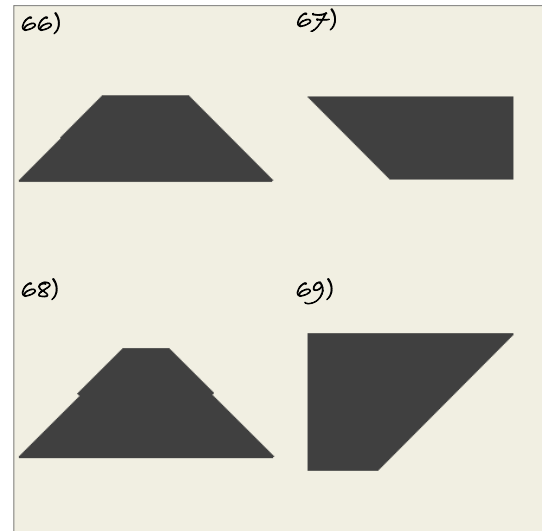


◀ Figure 2  
*The thirteen's home*  
2500x2500 giclée  
reworked from an  
original picture  
from the site  
geocities/tangmath



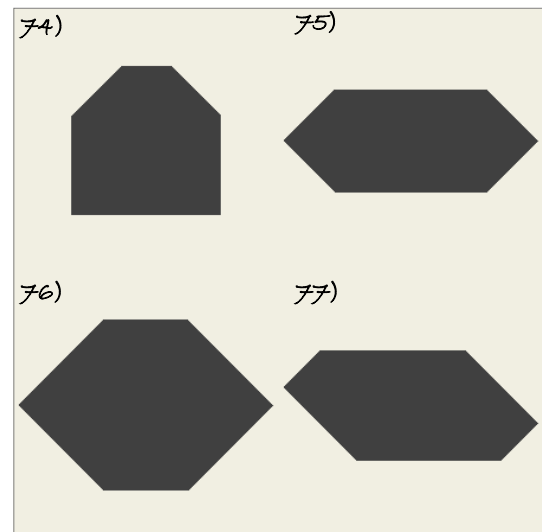
◀62-65  
The very basic  
convex polygons:  
N. 62 triangle,  
N. 63 square,  
N. 64 rectangle,  
N. 65 parallelogram.

66-69►  
Four trapezes



◀70-73  
Four pentagons

74-77►  
Four exagons



## Chapter 3

Here is the answer!

Three silhouettes in the previous page (N. 68, 70 and 73) look almost like convex figures, but they are not. They have small notches and bulges (not due to a bad quality of the images) which actually make them concave figures. These features are due to matching incommensurable sides proportional to 1 and  $\sqrt{2}$ . Such point will be analyzed further in the section on paradoxes.

All kidding aside, the above arguments suggest the investigation of the class of the “almost convex patterns”.

Lets suppose to fill in the notch of a silhouette like N. 78 in order to get a convex figure as small as possible. The fuchsia coloured extra piece has a surface area which is 2.6% of the total silhouette surface. That percentage,  $X$ , is a measure of how far the pattern is from being convex.

The pattern N. 79 has  $X=5.2\%$ .

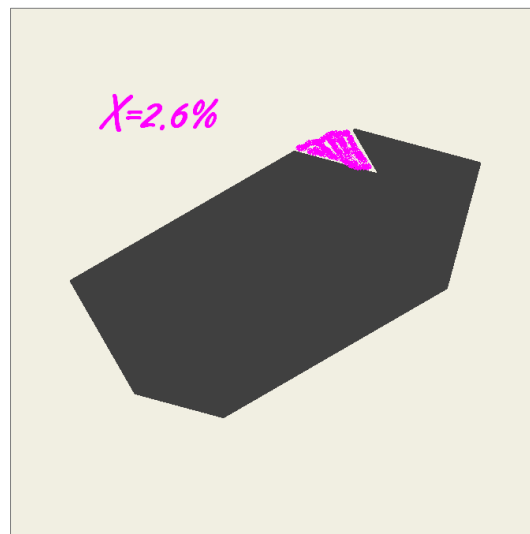
The patterns N. 68, 70 and 73, disguised as convex figures in the previous page, have still lower  $X$  values: 1.5%, 2.3% and 0.7%, respectively.

We can define “almost convex patterns” those having  $X$  less than e.g. 10%. Most Tangram silhouettes are not so compact; they have  $X$  ranging between 20% and more than 50% (see for instance silhouette N. 89).

On the other side, we can built silhouettes with  $X$  as small as we like (e.g. 0.1%). It can be done as in N. 86, 87 and 88 simply by taking two sides of a convex figure apart. By reducing the gap between the two parts you can reduce the  $X$  value. Of course, these figures can be nice to be view but they do not add any to the convex figures set as a brain teaser.

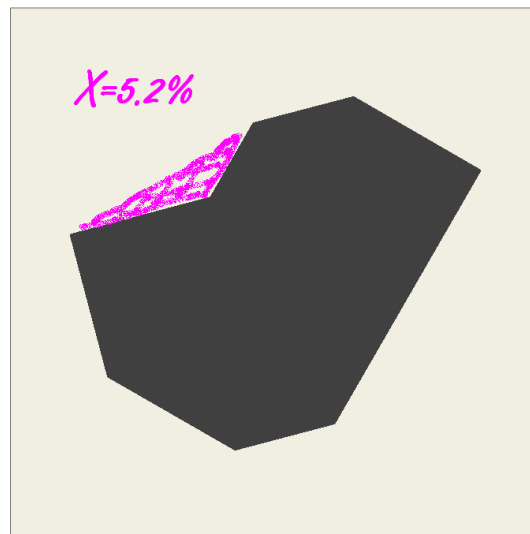
78►

Almost convex pattern with a 2.6% notch

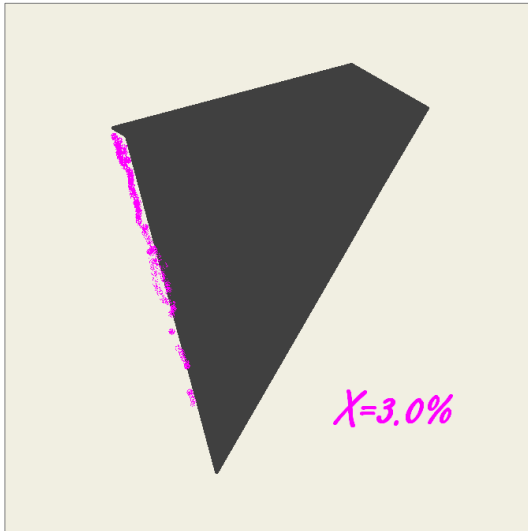


79►

Another almost convex silhouette

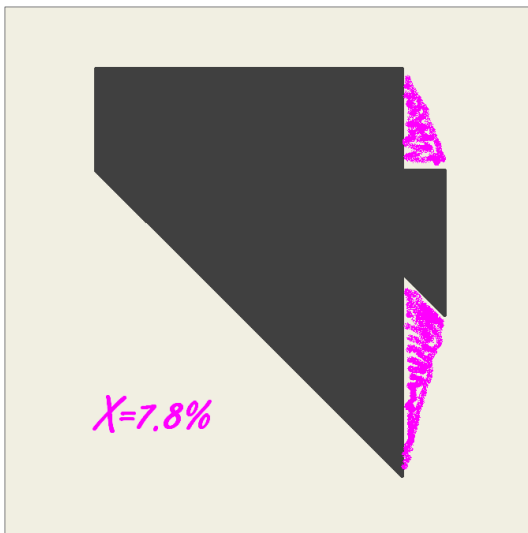
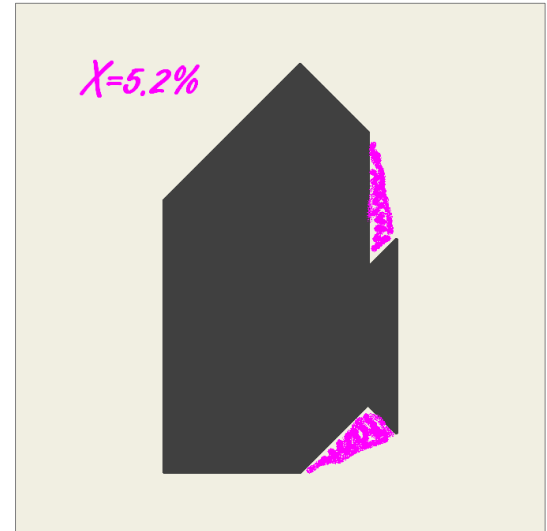






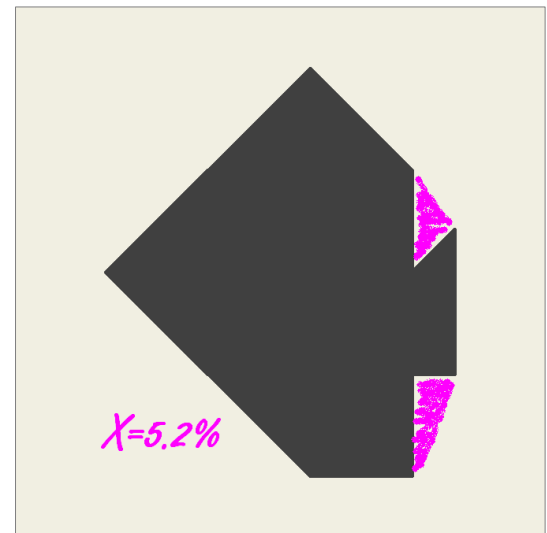
◀80

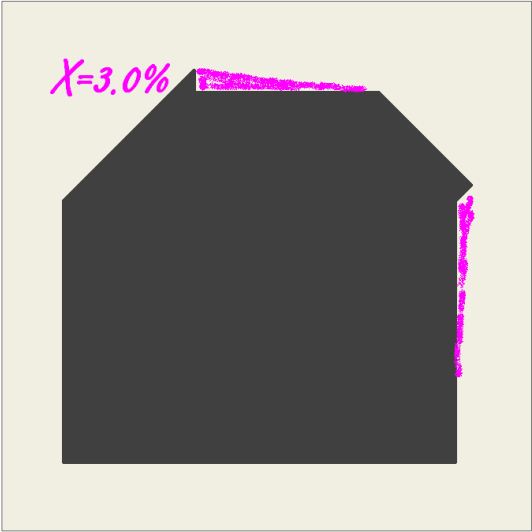
81▶



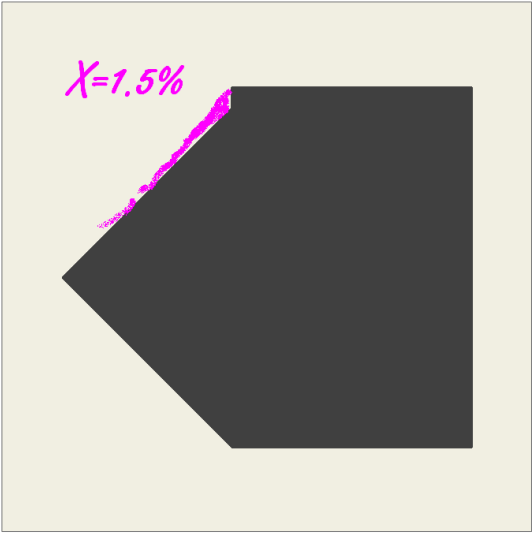
◀82

83▶



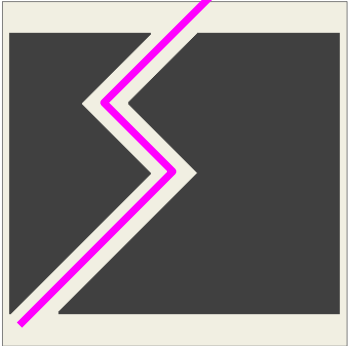
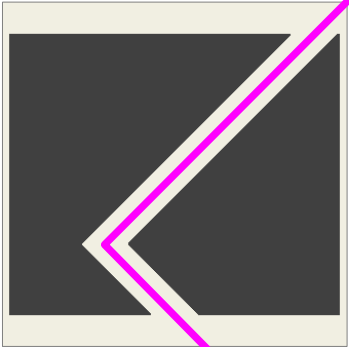
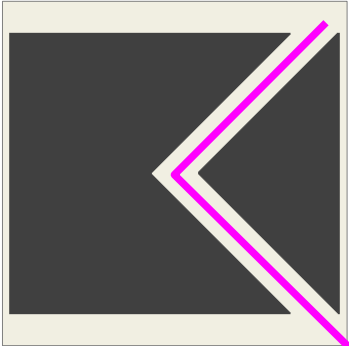


◀84



◀85

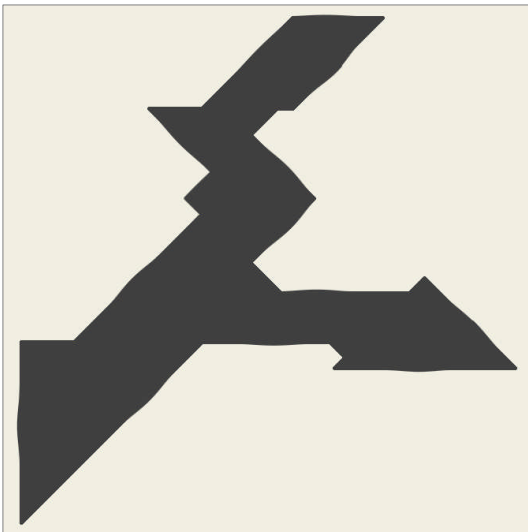
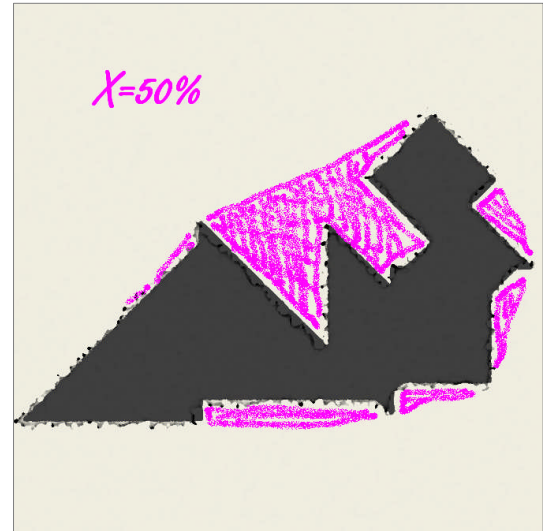
From top to bottom  
86-88▶  
Lightning



At the opposite side, there are patterns with few or no matched vertices at all. Despite their large X values, they can be hard to solve as well. As a matter of fact, you could be dazed by highly segmented patterns, as if you need more than just seven pieces to get all the features. That is the case of the patterns in this page.

89►

Segmented pattern  
in the fashion of  
the sculpturer  
Henry Moore

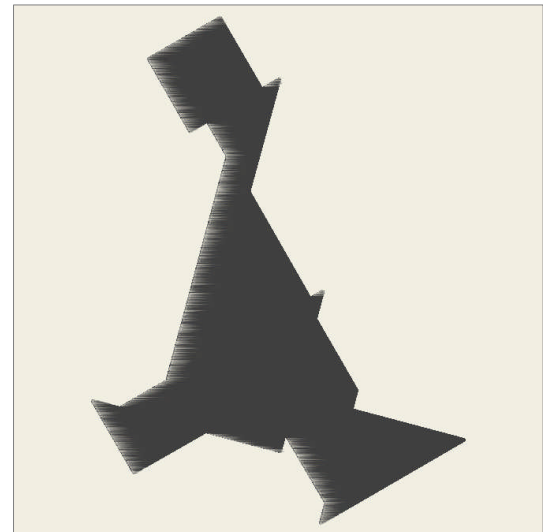


◄90

Segmented pattern  
after *Unique Forms  
of Continuity in  
Space* (1913) by  
Umberto Boccioni

91►

Walking



# 4 Arabesques

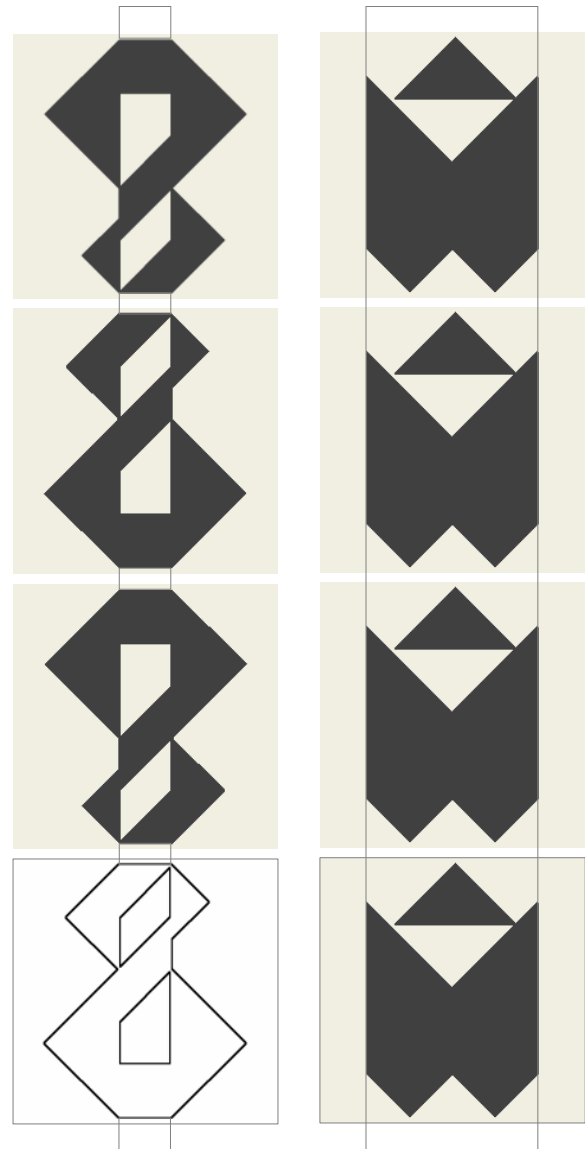
## *and geometrical decorations*

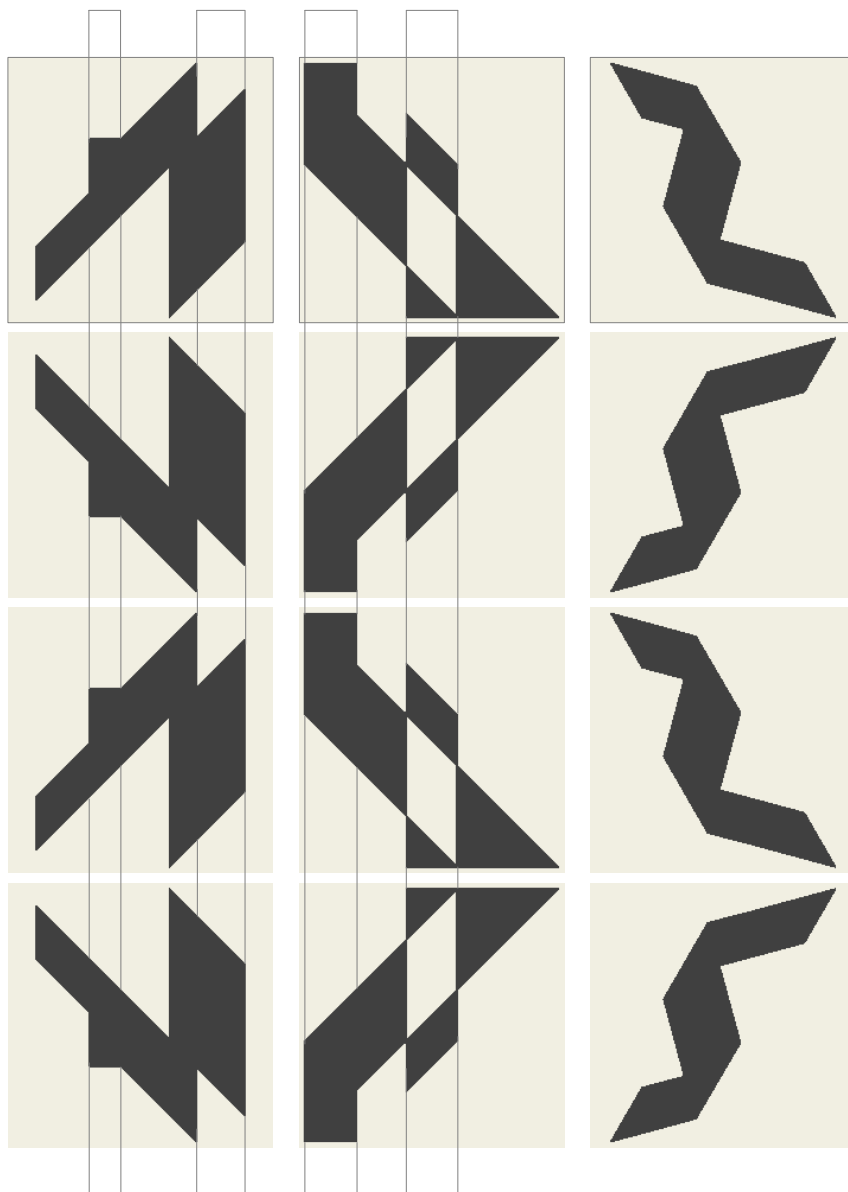
Tangram patterns are suitable for doing nice geometrical decorations. They often suggest by themselves some direction for repeating their geometric forms. For instance, the eight-shaped pattern N. 92 suggests the twist around a central rod.

Many arabesque can be obtained. Of course, any repetition unit is a puzzle to be solved.

92 (left) ►  
Number eight

93 (right) ►

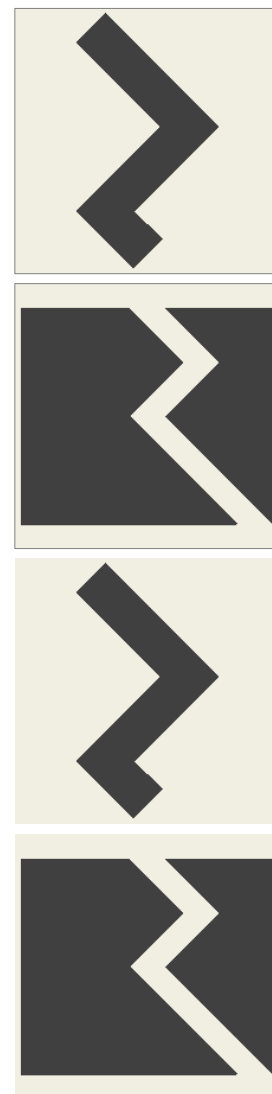




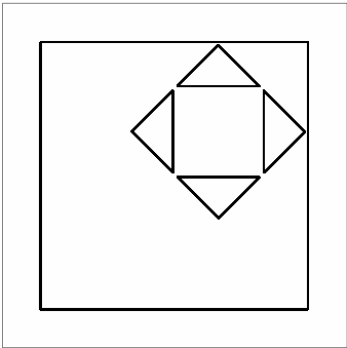
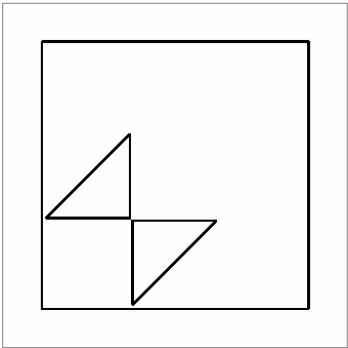
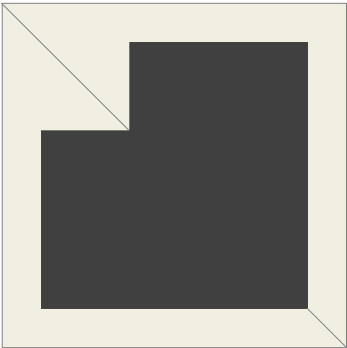
◀ 94 (left)  
95 (middle)  
96 (right)

97▶

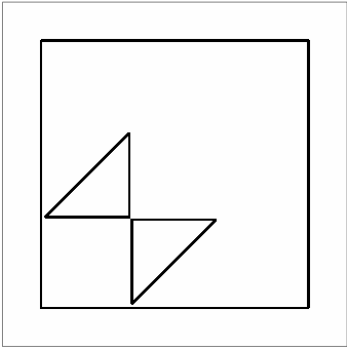
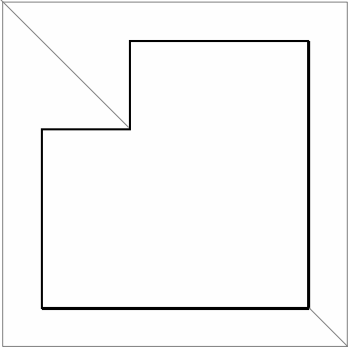
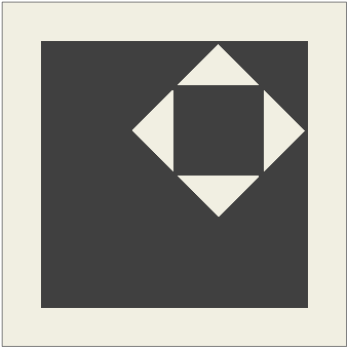
98▶  
same of N.88  
reversed



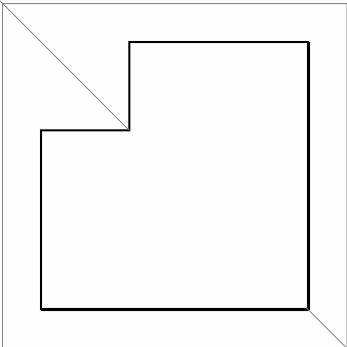
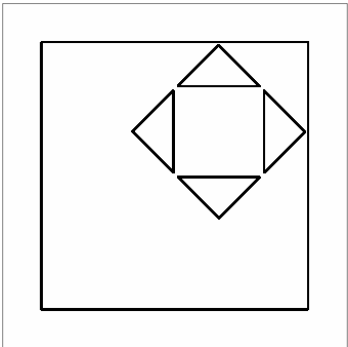
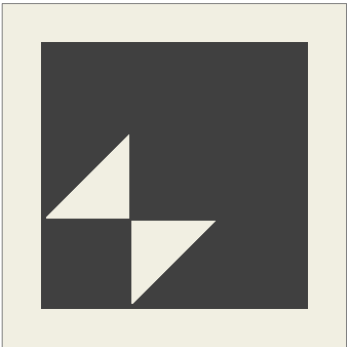
99▶



100▶



101▶



## 5 Paradoxes and fallacies

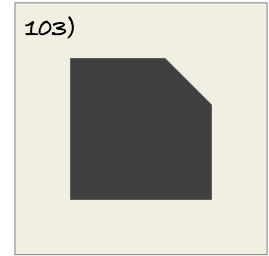
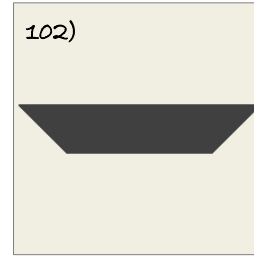
One of the most famous source of Tangram figures is “The eighth book of Tan” by Sam Loyd, published in 1903. It is a joke on Tangram history. According to the quizzical tone of the book, the author presented also few patterns which would seem not to be true Tangrams (see, for instance, the introduction of 1968 Dover edition by Van Note). Among them, there are the convex silhouettes reported above as N. 102 and 103.

The former is presented in Loyd’s book as a milk bowl with two kittens around (the same scene I reported at page 13). The latter is a clipped square. After Wang and Sung convex figures counting (see page 15), we can definitely say that both N. 102 and 103 cannot be obtained with a Tangram set.

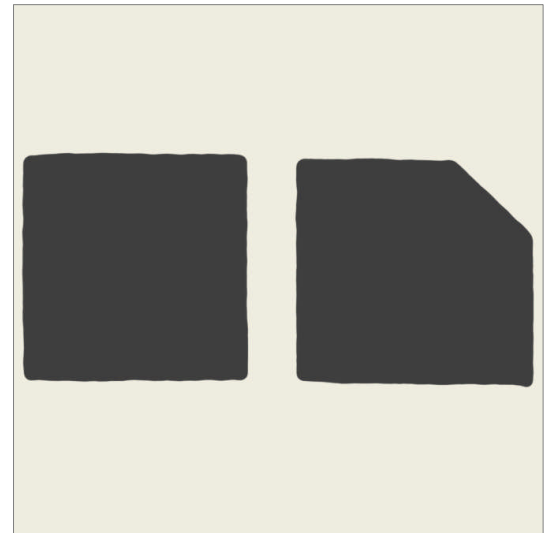
So where is the trick?

A solution is that Loyd was kidding (and cheating) by using two sets of Tangram, as shown at page 59.

But in the case of the clipped square, I think Loyd didn’t really cheat. He actually presented a sequence of eight shapes ending with two figures, similar to those in N. 104. The shape on the left is similar to N. 63, the square, and the shape on the right is similar to N. 103. So it’s natural to think them as built using two separate Tangram sets. On the contrary, it is possible to reproduce both the shapes of silhouette N. 104 with only one set.



Sam Loyd, *The eighth book of Tan*



### ▲ 104

Sam Loyd presented a sequence of eight shapes ending with two figures like these. He wrote: “The seventh and eighth figures represent the mysterious square, built with seven pieces: then with a corner clipped off, and still the same seven pieces employed.” How is it possible?

## Chapter 5

Paradoxes is a classical topic in Tangram literature. A Tangram paradox is an apparent dissection fallacy: two figures composed with the same set of pieces, one of which seems to be a proper subset of the other. One famous paradox is that of the two monks, one with (N. 105) and the other missing a foot (N. 106).

Paradoxes could be divided into two main groups: the first one composed by well matched figures in which one image has exactly the same number of vertices of a subset of the other image (as the monks, N. 105-106 and 107-108); the second one composed by nearly matched figures, in which the two images actually differ for small spikes (as the two buildings, N. 109-110) which are not evident in small, handmade or reworked pictures.

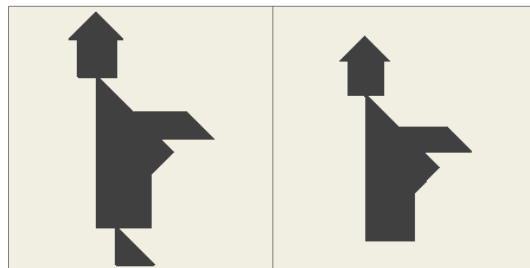
Some almost convex figures can be considered as paradoxes of the second group (e.g. N. 115-116 which have been presented as N. 71 and 73 among the convex figures at page 19).

The devilish Loyd even proposed a triple paradox (N. 117-119): "With the figures of the three cabinet organs we reach that borderland of mystery in the black art which can be only solved mathematically.

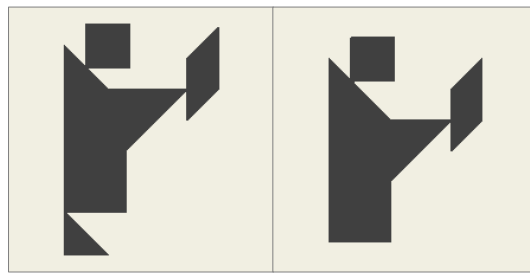
... yet the last one shows a folding lid which calls for an extra piece!"

Indeed, here the mathematicians Wang and Sung cannot help us, since the figure are clearly not convex. Nonetheless, I think Loyd was kidding again since N. 117 and 118 seem not true Tangrams. But who knows? I cannot prove it.

105, 106 ►  
Two monks, with  
and without a foot



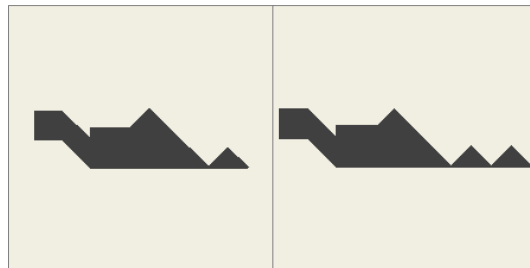
107, 108 ►  
Two more monks,  
with and without  
a foot



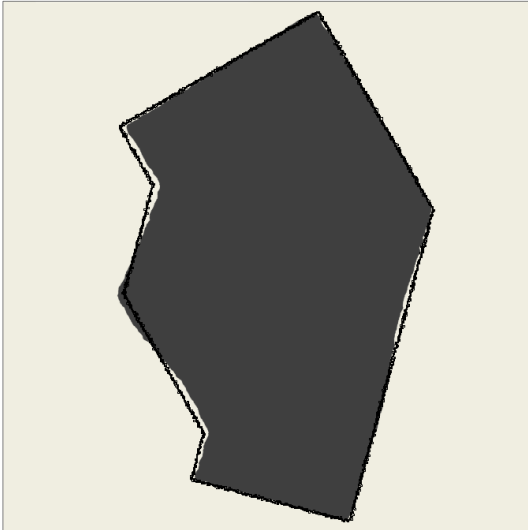
109, 110 ►  
Buildings with  
closed and open  
stair



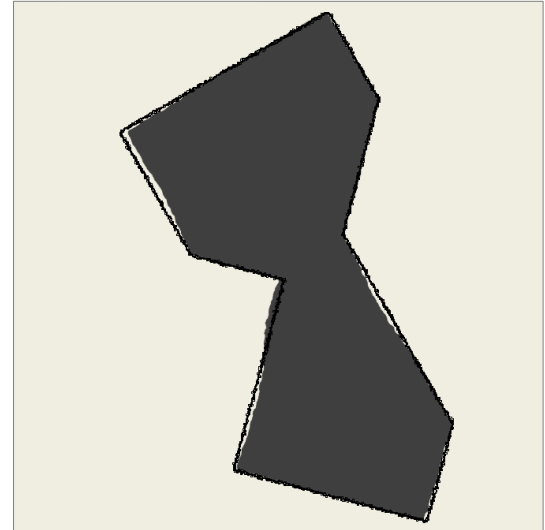
111, 112 ►  
Dragons with short  
and long tail



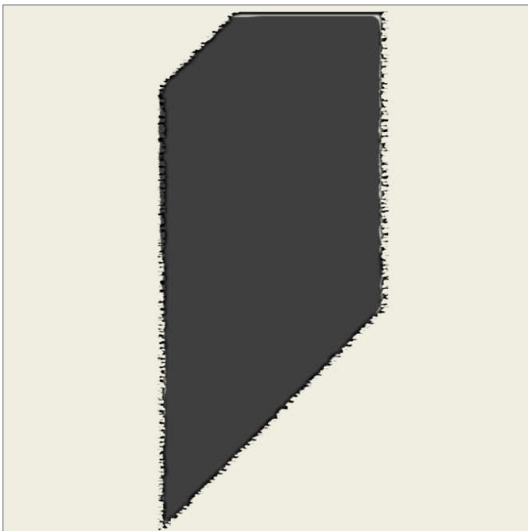




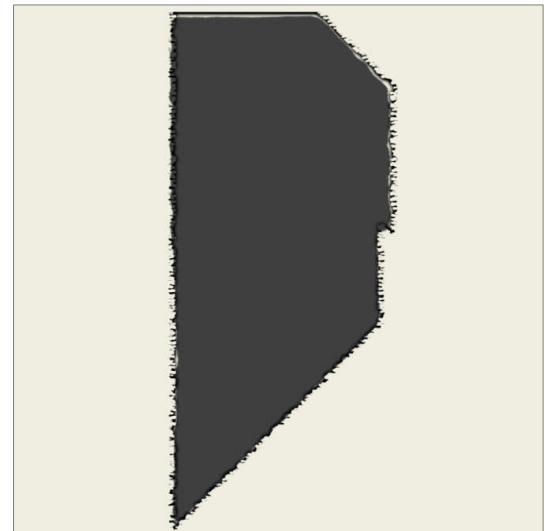
◀113  
Inflated



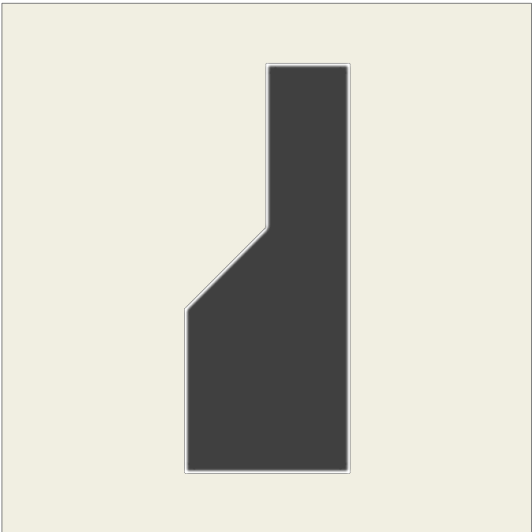
114▶  
Deflated



◀115  
Hump on the right.

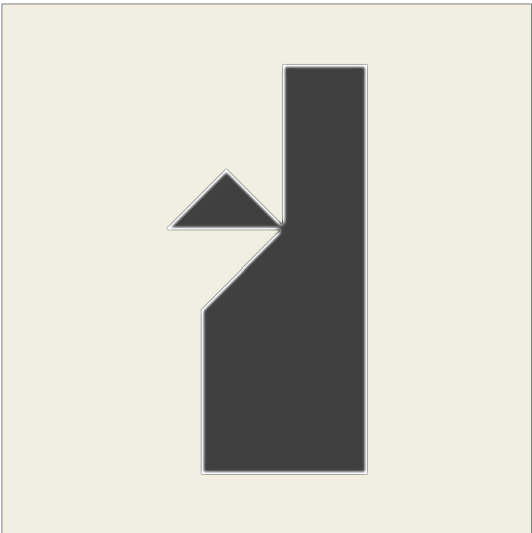


116▶  
Hump on the left.  
"What hump?" as  
Igor-Marty Feldman  
replied to Doctor  
Frankenstein asking  
for which side the  
hump on his back  
was located, in  
*Young Frankenstein*  
by Mel Brooks



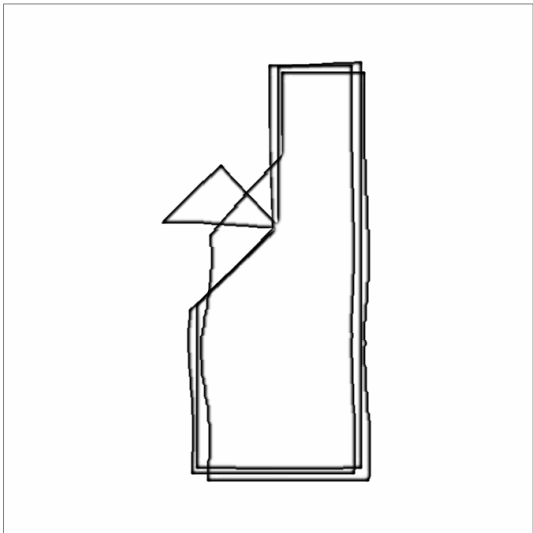
◀117  
A cabinet organ  
from Loyd's book. Is  
it a true Tangram?

118▶  
A second cabinet  
organ from Loyd's  
book. Is it a true  
Tangram?



◀119  
A third cabinet  
organ with a folding  
lid from Loyd's  
book. Is it a true  
Tangram?

Figure 3▶  
We reach that  
borderland of  
mystery in the  
black art...



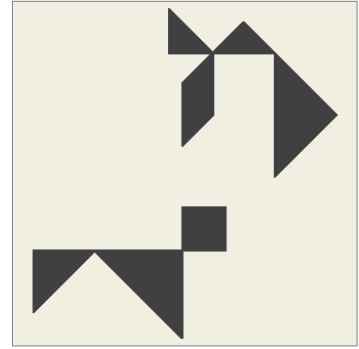
## 6 Double Shapes

*when one is better than two*

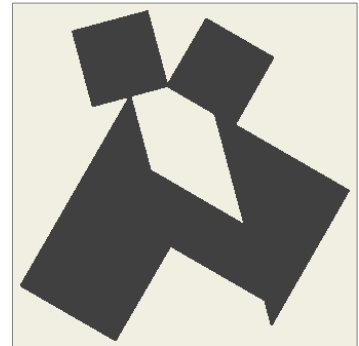
Tangram figures can be used to make scenes with more characters and objects, as at page 13. Of course, more Tangram sets are needed. Quite unusual is what I call “double shapes”, that is to say one-Tangram-set patterns which are separated into two distinguished parts to form a scene. At a first glance, they seem to need two Tangram sets to be solved; on the contrary, they need just one. In particular, in some cases they cannot be solved at all using two sets.

We have already met an example of such double shapes, the Loyd’s clipped square (N. 104). Also any shapes in which all the seven pieces are distinguished, as in the picture on the right (N. 120), cannot be solved with two Tangram sets. That’s again a paradox to me.

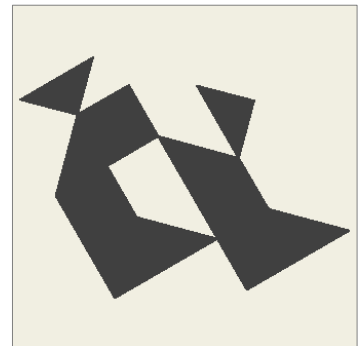
120 ►  
Wild  
horses

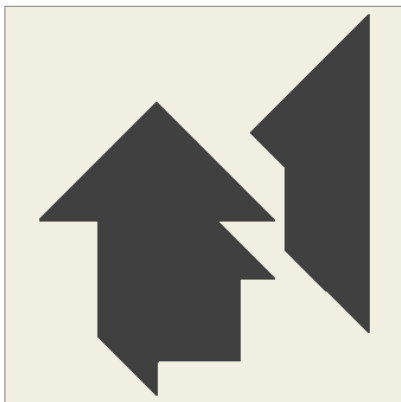


121 ►  
The kiss

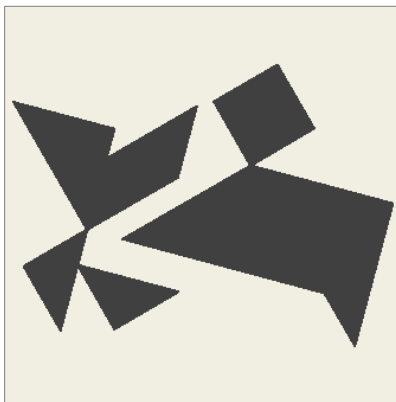


122 ►  
Dancing

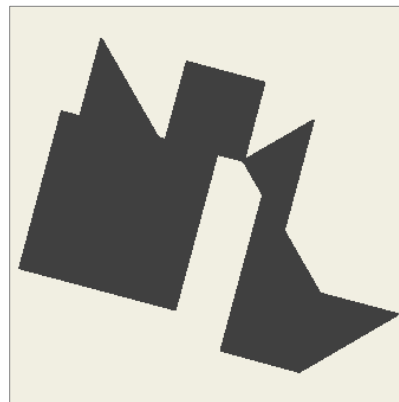




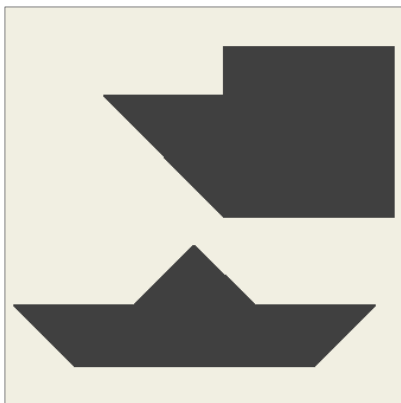
▲ 123  
People



▲ 124  
Mom and pram



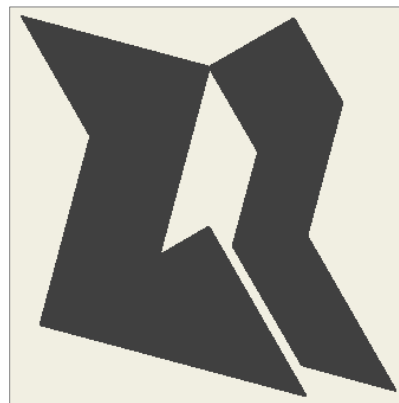
▲ 125  
Aquatic birds



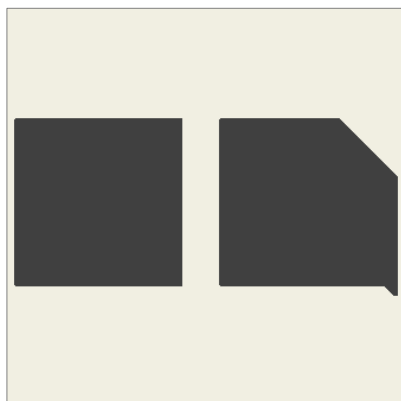
▲ 126  
Ships



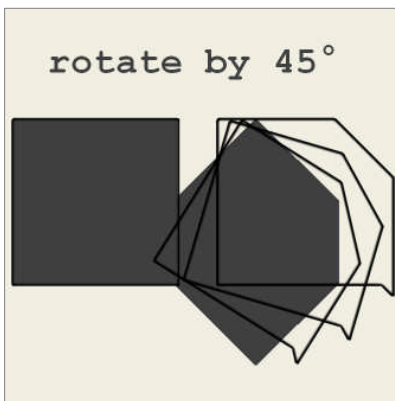
▲ 127  
Hug



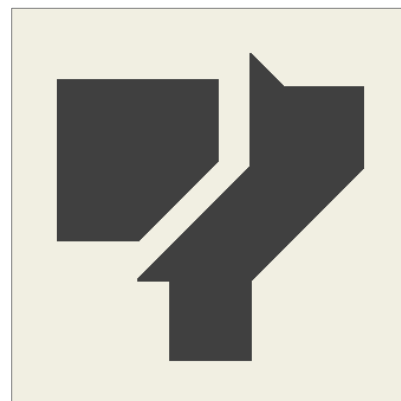
▲ 128  
Swans



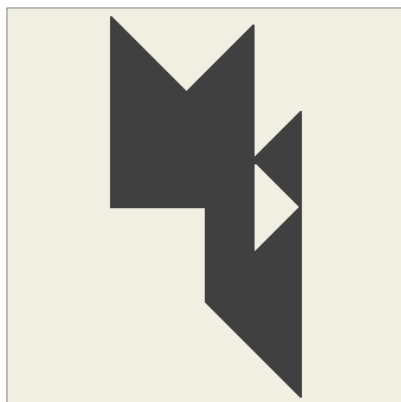
▲ 129  
The square and Loyd's clipped square, same of N. 104, for comparison with N. 130.



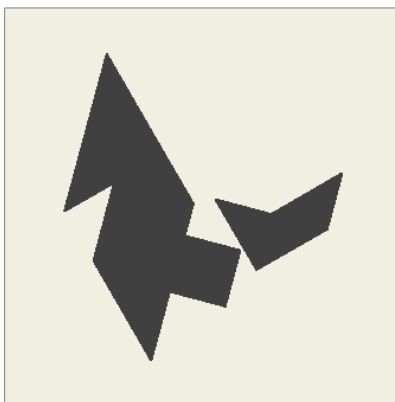
▲ 130  
As N. 129 rotated by 45°.



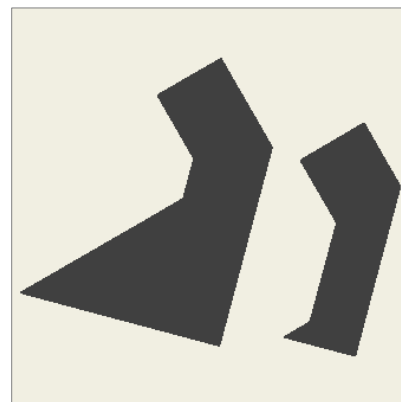
▲ 131  
Trying to eat the Loyd's clipped square.



▲ 132  
Cat and mouse



▲ 133  
The kick



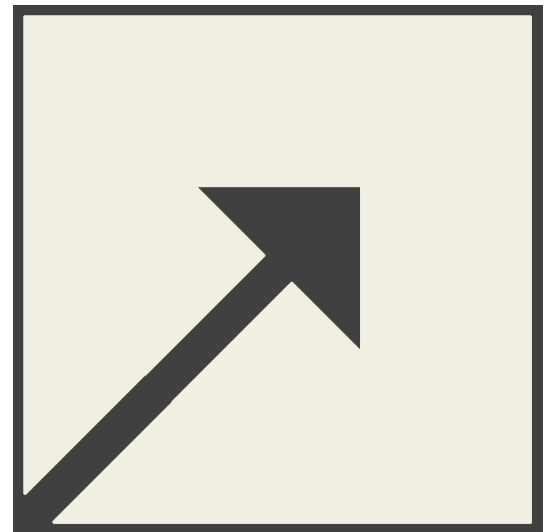
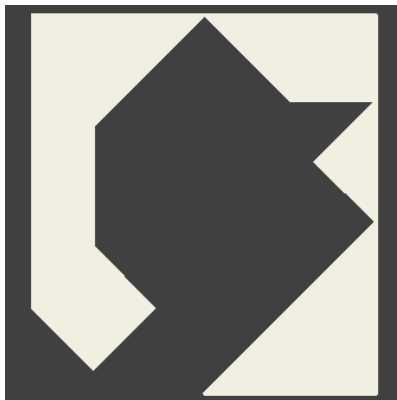
▲ 134  
Dancing together

## 7 Inverted Colours *and visual illusions*

Visual illusions are characterized by visually perceived images that differ from objective reality. Our brain processes information gathered by the eyes and, sometimes, the perception does not correspond with a physical measurement of the stimulus source. In particular, cognitive illusions could arise by interaction with assumptions about the world, leading to "unconscious inferences".

Indeed, Tangram itself is a visual illusion, thinking to figures as those in "like a Tangram world" section, as well as those in the "Paradoxes" section. But, there is some more.

We are used to think to Tangram silhouettes as black figures on a pale background, as "Chinese shadows"; and up to here we have mainly followed such an use. Now, look at the arrow on the right and at the figures in this section. They all are Tangrams, and, I'm sure, at a first glance most of us percept the pieces as black, as usual. So doing we will find some problems in solving them! Instead, the silhouettes of this section have been displayed by inverting the colours (all but one).



▲ 120  
Arrow

◀ 121  
A man looking at the arrow



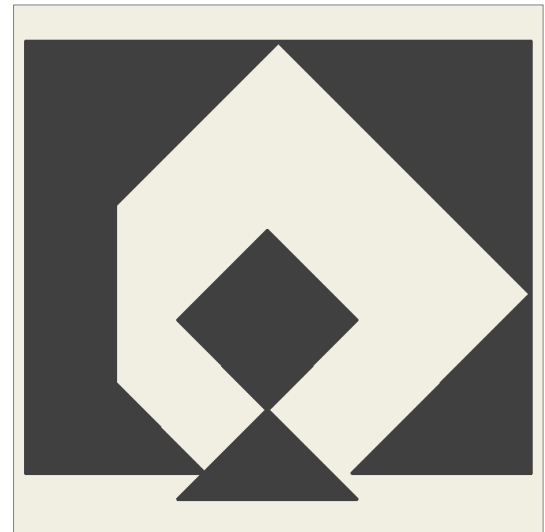
◀122  
A cat



123▶  
A woman at the  
window



◀124  
A dog face



125▶  
A man at the  
window

Full many a gem of purest ray serene,  
The dark unfathom'd caves of ocean bear:  
Full many a flow'r is born to blush unseen,  
And waste its sweetness on the desert air.  
Thomas Gray, *Elegy Written in a Country Churchyard*

## 8 Akin Abstract Art

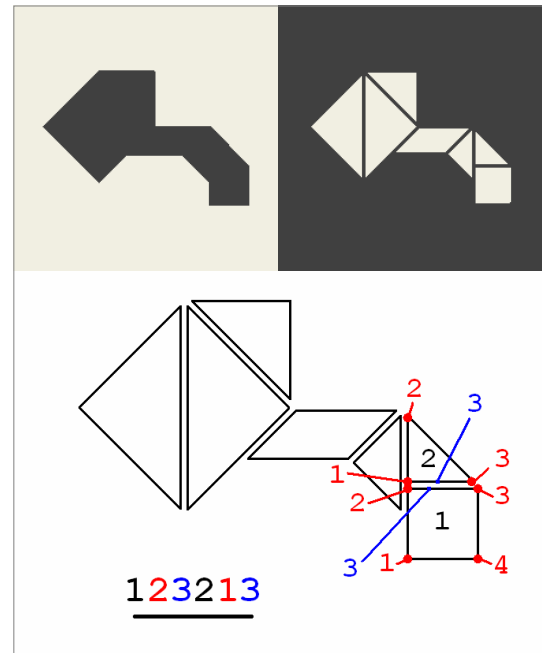
### *Rheograms as Generative Art*

It has been noticed how Tangram is the opposite of a jigsaw puzzle, which allow just one configuration using a lot of pieces. As few as the seven Tangram pieces allow a multitude of different configurations. But, how many? Probably more than one billion, neglecting unessential displacements of the pieces.

I tried to make some inference and simulation to estimate such a number. To do that, I developed a computer program which is able to randomly generate proper Tangram patterns. The software is based on a specific format which basically contains a sequence of 6 sets of 6 digits. Each set describes the connection of sides and vertices of a couple of tans. Referring to the example in fig. 7, the set 123213 indicates that the square (piece n.1) is connected to the first small triangle (piece n.2), by linking the vertex n.2 of the square to the vertex n. 1 of the triangle, and by matching the edge n.3 of the square to the edge n.3 of the triangle. A complete example of the format is the number sequence in the bottom of the front cover.

The program generates random sequences of these digits and discards those corresponding to configurations where some piece superposes on another. A version of this software is available for free at [www.tanzzle.com](http://www.tanzzle.com).

By running the program, some estimates of the number of allowed configurations can be done. Doing that, I was fascinated by the flow of the Tangram configurations generated on the computer screen. So, I dismissed the counting problem and focused onto the "Generative Art", which, I discovered, is a well-assessed field of the Arts.



▲Figure 7  
Format for pattern description



Generative art has been defined by Philip Galanter as “any art practice where the artist creates a process, such as a set of natural language rules, a computer program, a machine, or other mechanism, which is then set to motion with some degree of autonomy contributing to or resulting in a complete work of art.”

I made several programs which I called “Rheogram”, from “rheos” which means “flow” in Greek, as running the program is a metaphor for the flow of time.

A rheogram displays more tangram patterns at a while (either silhouettes or wireframe solutions), and continuously updates in an endless sequence.

First, I used a FIFO (First In First Out) queue of 33 elements. At each clock (one second or more), the figures shift their positions, the oldest configuration disappears and a new pattern appears at the first place, according to the scheme reported in fig. 8.

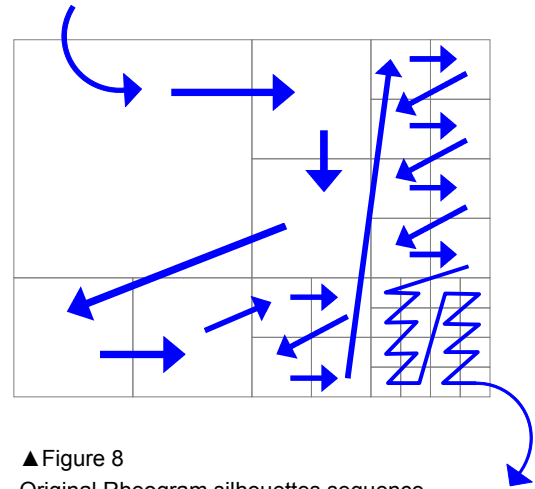
The sequences of figures is unrepeatable on the screen at a very high degree of confidence. Each tangram pattern is displayed 33 times on the screen and lost for ever. A print screen image as in fig. 10 is a shot for memory’s sake.

A specific pattern can be sometimes propagated all over the screen (as in fig. 11), but after a while the update will restart.

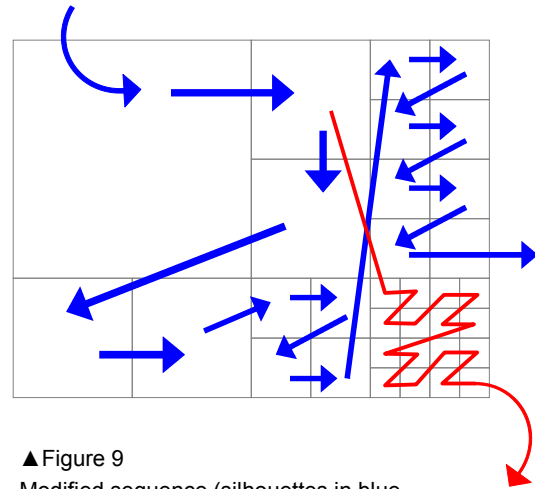
The colour choice and mixing are essential features of those works.

I did also a version for puzzle players with a queue of 17 silhouettes and 16 solutions at the same time. At each frame, a new silhouette appears on the left top side and the previous 16 silhouettes shift according to the scheme in fig. 9. The corresponding 16 solutions shift according to the red line sequence. As a matter of fact, the solution of the last generated silhouette is not available from the print screen. Shots of this version is reported in figs. 12-14. The solutions to the last generated silhouettes are reported in chapter 10.

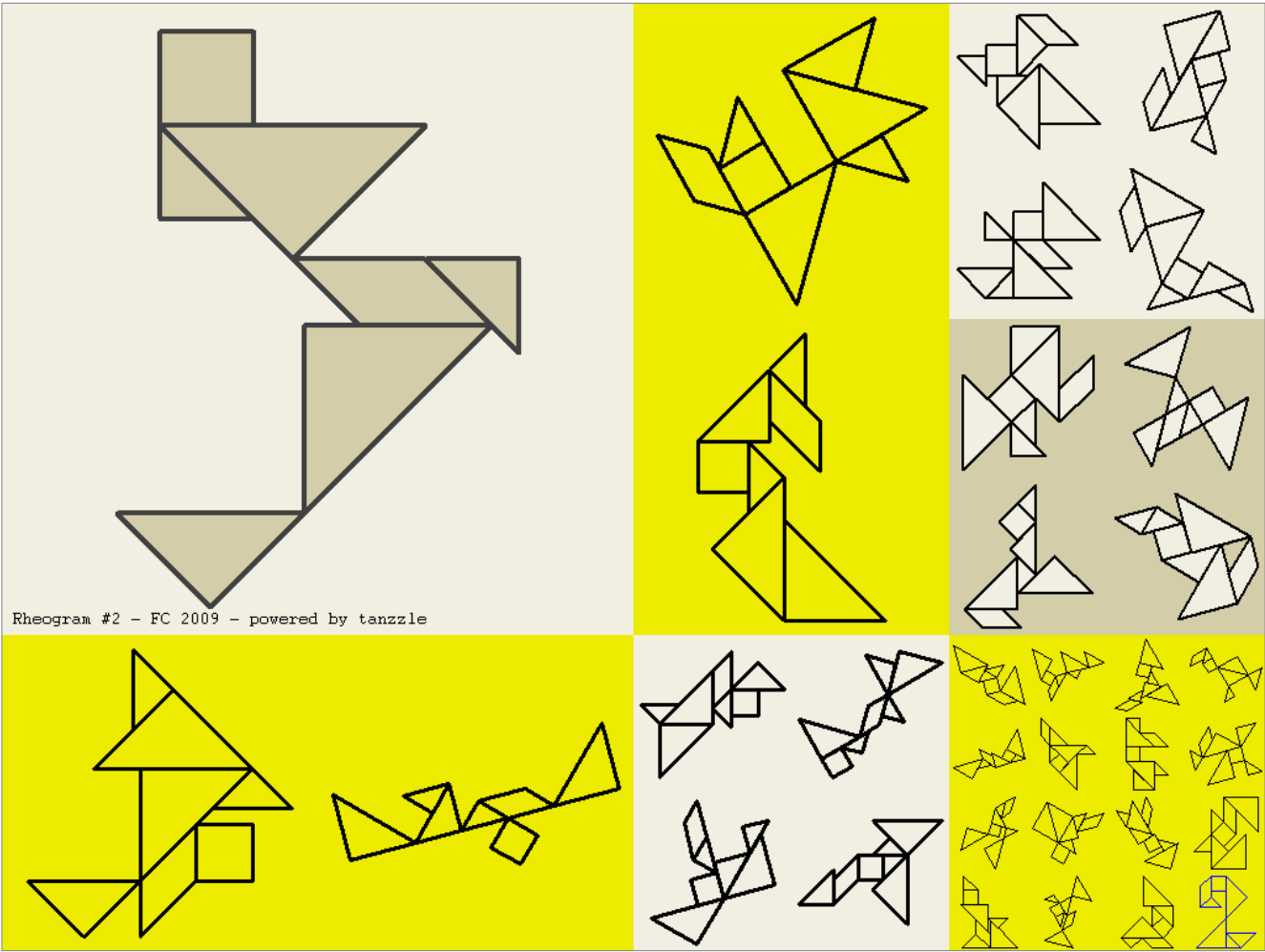
I found that the inexorable rheograms’ rhythm give anxiety to somebody, but it depends on his mood. By certainly, that is a way to see at new silhouettes, and it can happen to discover happy laughing faces too, as in fig. 13.



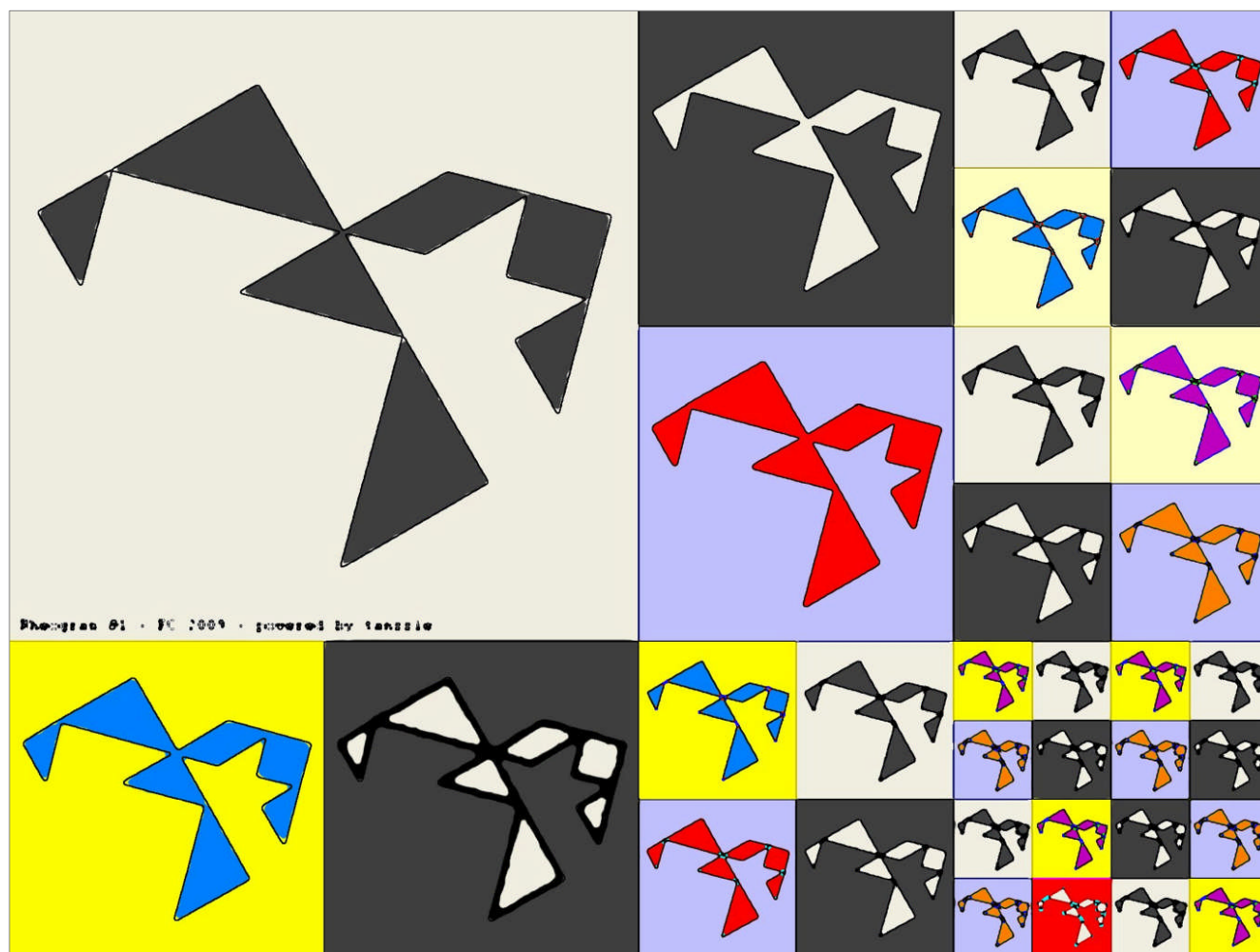
▲ Figure 8  
Original Rheogram silhouettes sequence



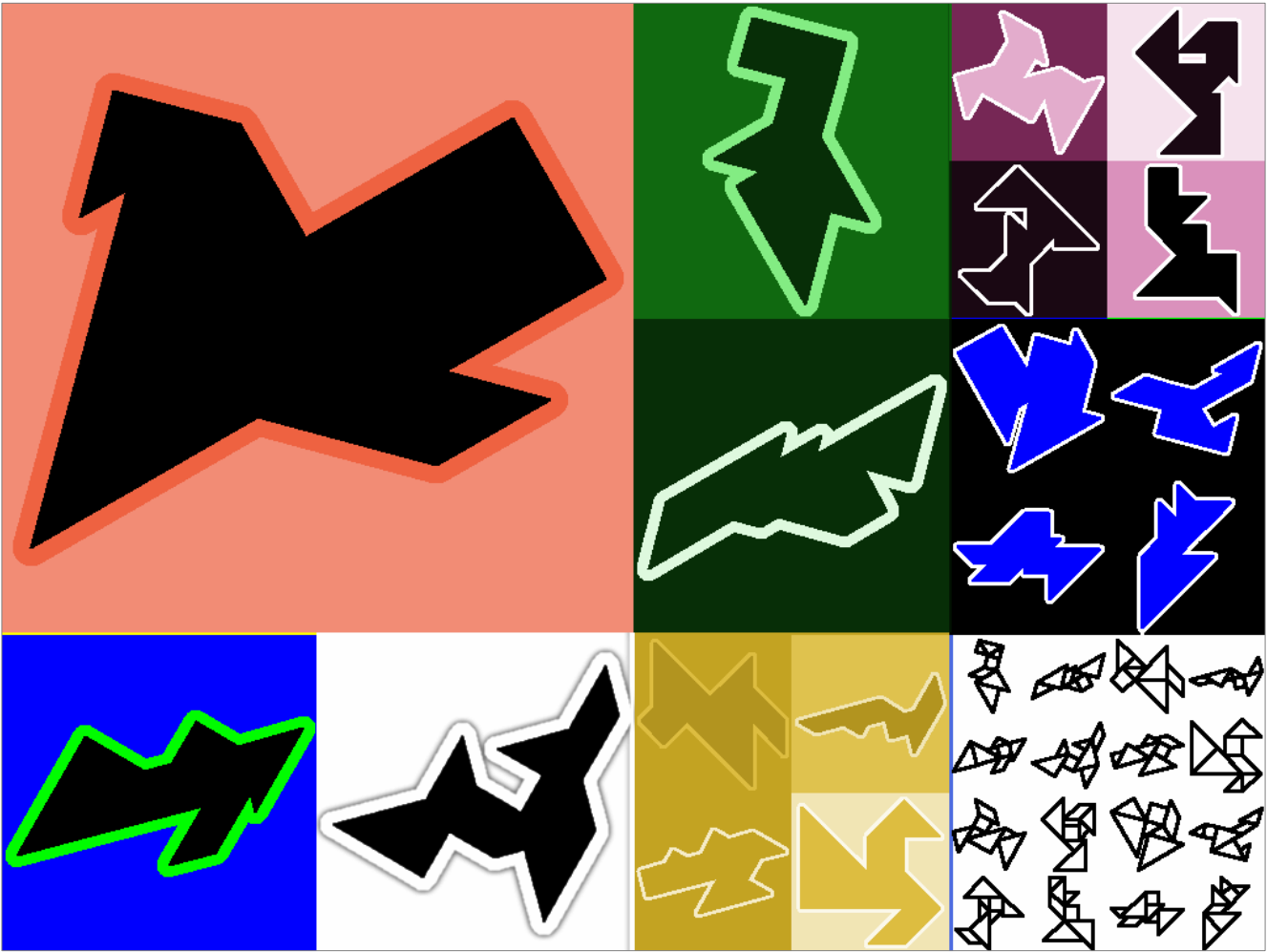
▲ Figure 9  
Modified sequence (silhouettes in blue,  
corresponding solutions in red)



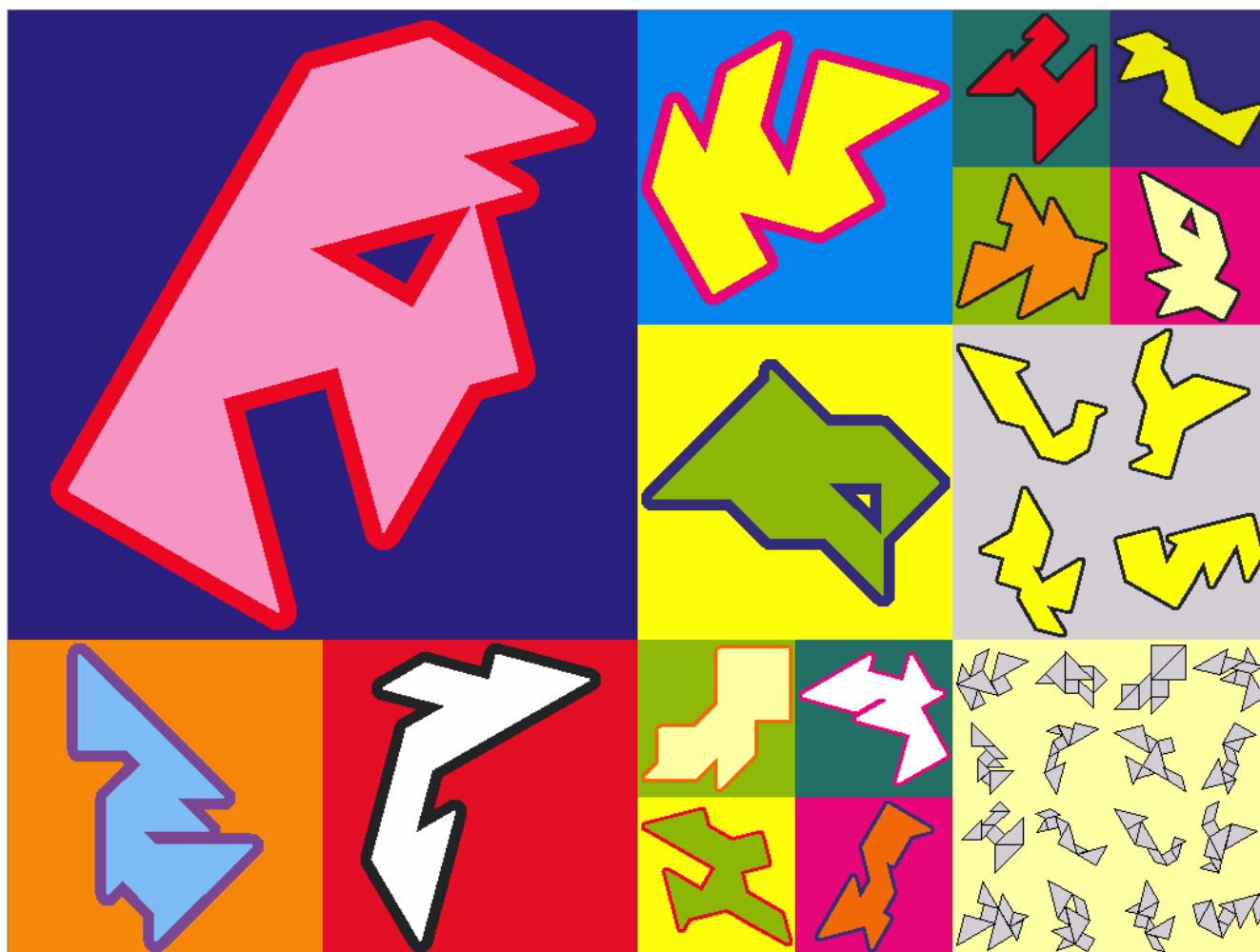
▲ Figure 10  
Rheogram #2 screen shot. Random patterns N. 126-158



▲ Figure 11  
Rheogram #1 screen shot. Random pattern N. 159



▲ Figure 12  
Rheogram #4 screen shot. Random patterns N. 160-176 and their solutions (except that of N. 160)



▲ Figure 13  
Rheogram #4 screen shot. Random patterns N. 177-193 and their solutions (except that of N. 177)



▲ Figure 14  
Rheogram #4 screen shot. Random patterns N. 194-210 and their solutions (except that of N. 194)

## 9 Sketches

*i.e. play tangram with a pencil*

Tangram addicted people love to handle good-shaped tangram sets, usually made with exotic wood essences. Nonetheless, Tangram is a mental, if not even spiritual, activity. The players should find (feel) the solution in their mind. Arranging the pieces should be a sort of “casting out nine”.

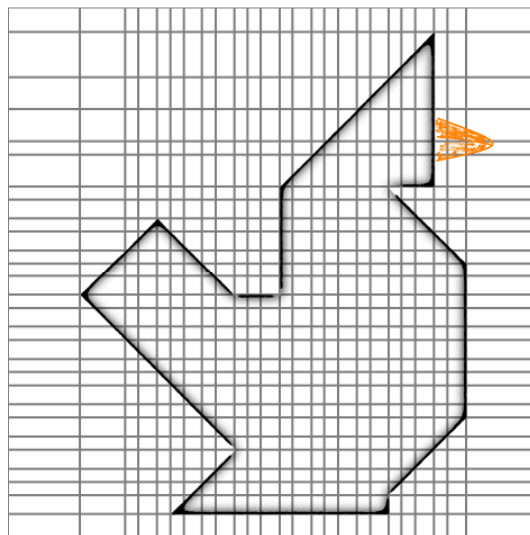
The ability to find the solution in our mind, without using the material pieces is, may be, the final level in the Tangram experience. Therefore, let me conclude this book with an unusual way of presenting Tangram shapes, yet visually impressive, which should help in developing such a skill.

The patterns look like children pictures on an arithmetic exercise book. The silhouettes are imprinted onto a grid which should suggest to look at Tangram as a dissection puzzle. The player does not have to handle and fit the pieces in order to get the images; on the contrary, he has to dissect the images in order to get the pieces, with the help of a pencil, at the beginning, and, at the highest level of training, only within his mind.

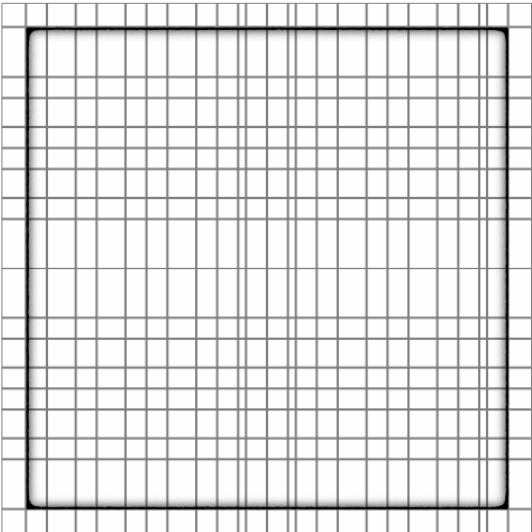
The grid is an aid for that purpose, not at all a hint to the solution of the puzzle. It isn't drawn in a casual way. It is formed by unequally-spaced perpendicular lines. The vertices of the tans have always to stay on some intersection point of the grid (see figs. 15-18).

The player sketches straight lines and tries to divide the figure in the seven polygons. A part of the segments should override the grid lines; the remainders should diagonally (45°) cross the grid lines. All the segments must have both the ends at some points of the grid. Train yourself and go back for solving in this way even the previous silhouettes. At the final level of training, you won't need the grid anymore!

*Take him and cut him out in little stars,  
And he will make the face of heaven so fine.  
W. Shakespeare, Romeo and Juliet, iii. 2.*



▲211  
Mother hen



◀Figure 15  
The sketch of the  
square silhouette

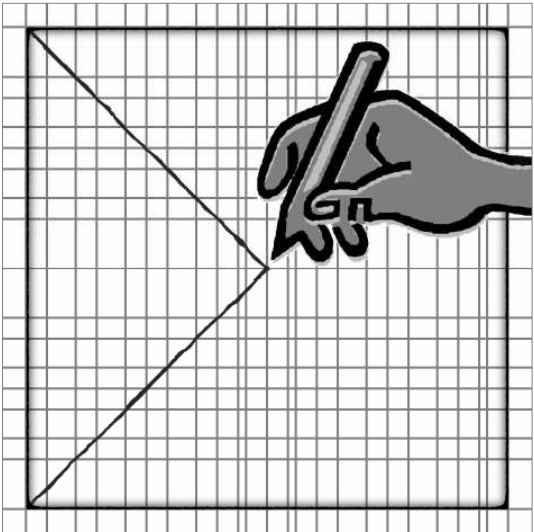
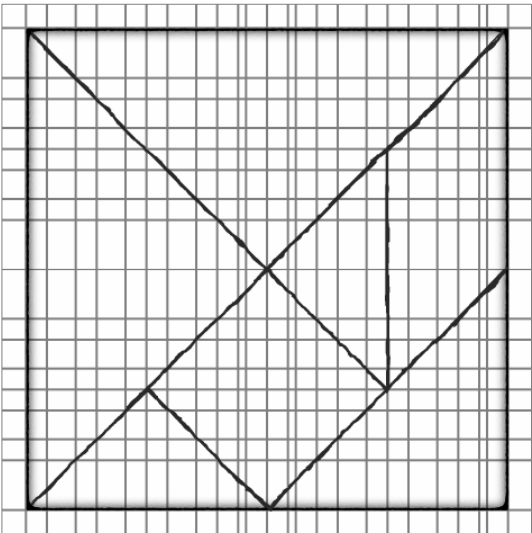


Figure 16▶  
Drawing the edges of  
the large triangle.



◀Figure 17  
Full solution.  
Each edge is either  
45° diagonal or  
overrides the grid  
lines. Each vertex  
is on a grid  
intersection point.

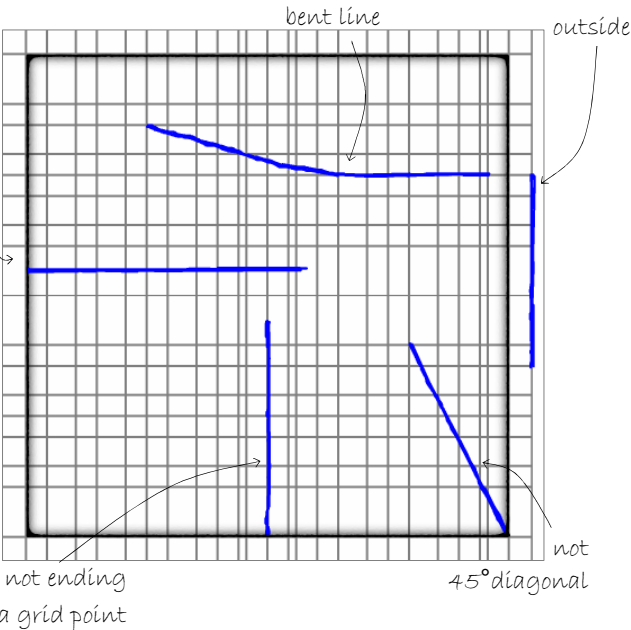
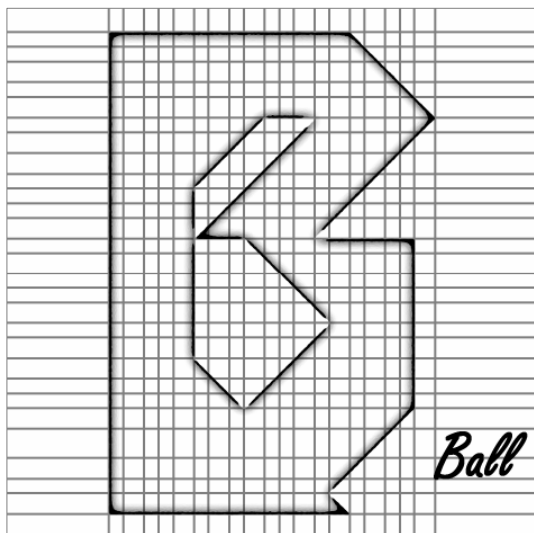


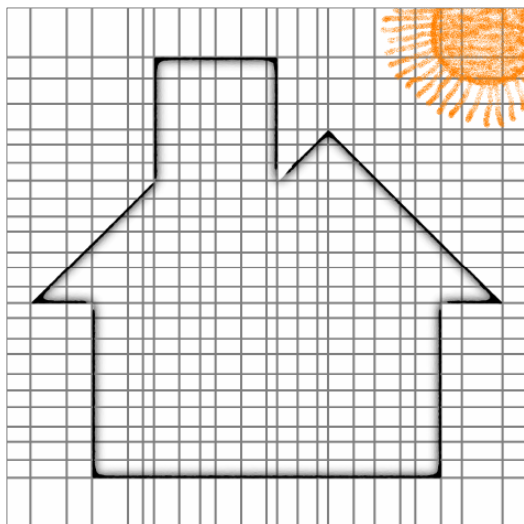
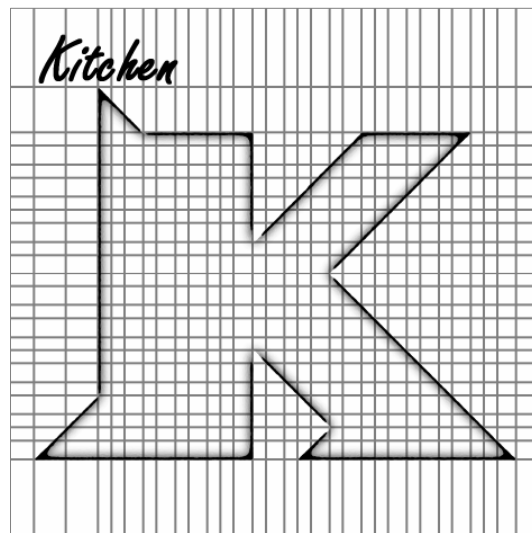
Figure 18▶  
Forbidden lines





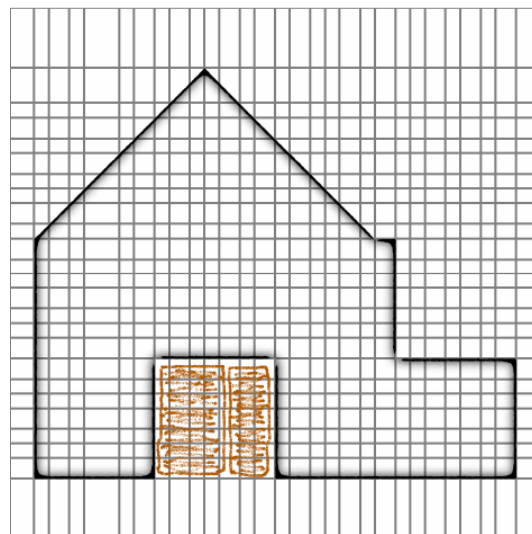
◀212  
B as Ball

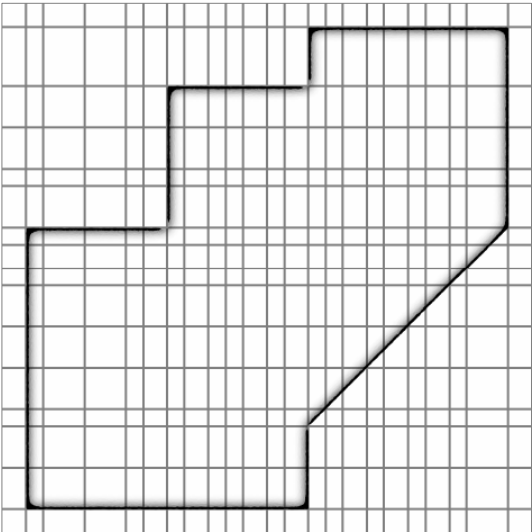
213▶  
K as Kitchen



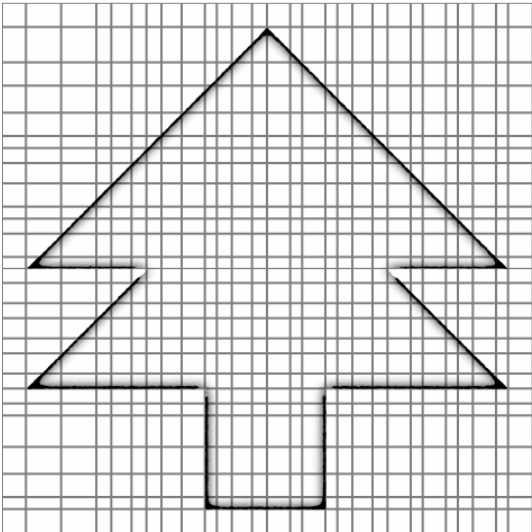
◀214  
Home

215▶  
Building

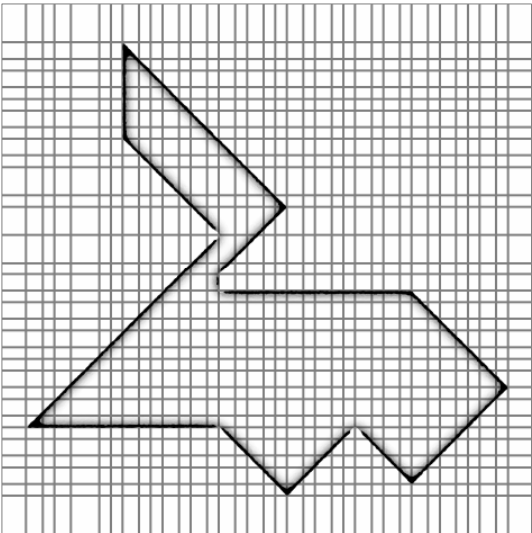




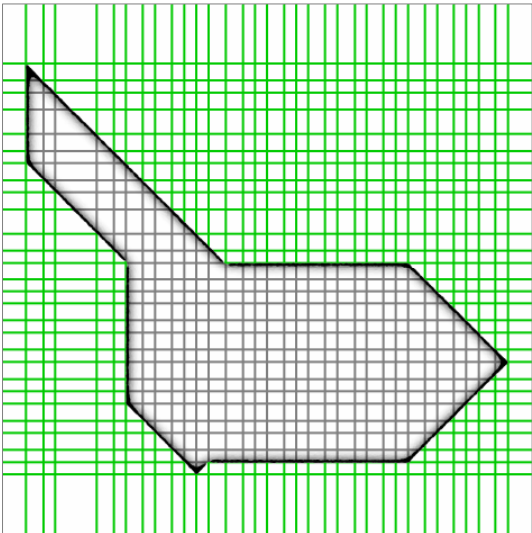
◀216  
Stair with uneven  
steps



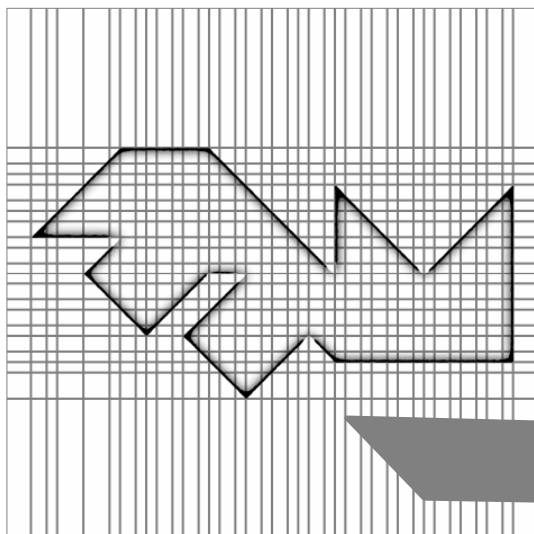
217▶  
Christmas tree



◀218  
Trolley

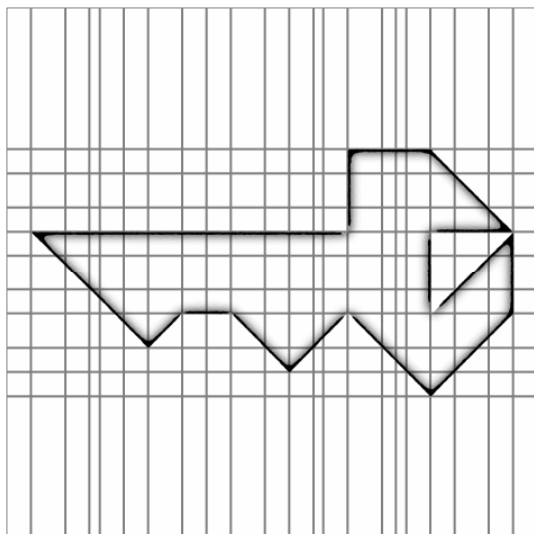
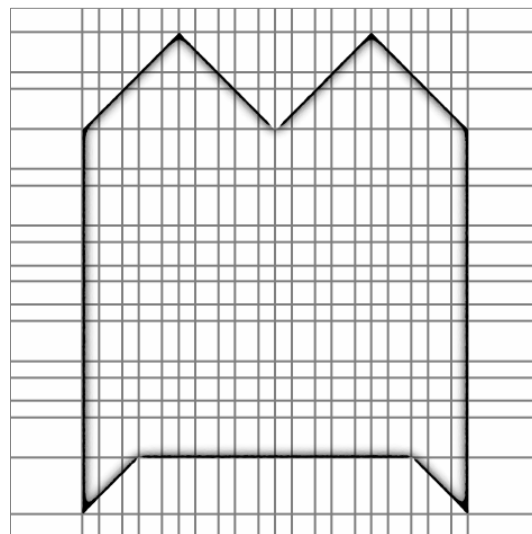


219▶  
Lawn mower

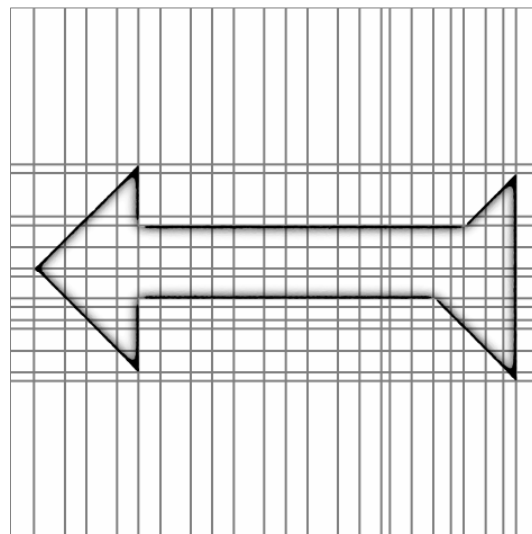


221►  
Sea cabins

◄220  
Kitten.  
Did the kittens N. 54  
and 55 leave any  
milk?

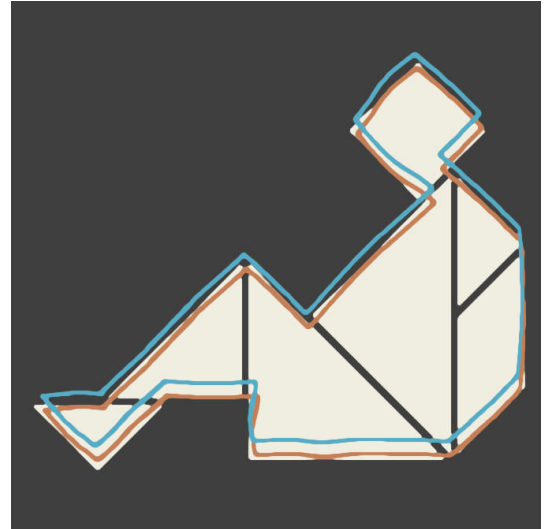


◄222  
Key.  
The key to the  
final level of the  
Tangram  
experience.

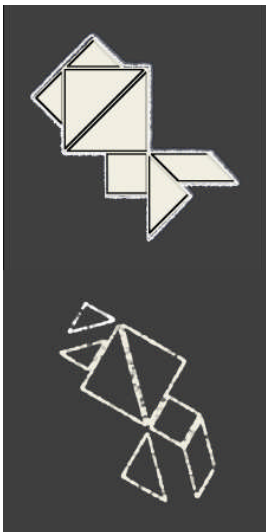


223►  
Arrow.  
Go back to the  
beginning and try  
to solve all the  
silhouettes of this  
book by dissection,  
in your mind.

# 10 Solutions



N. 0 ►  
Silhouette in the  
forewords page



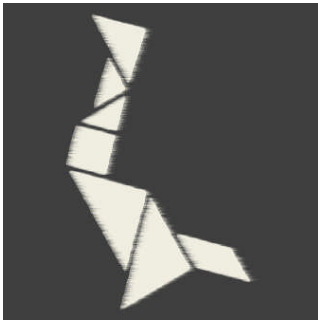
◄ N. 2

◄ N. 3

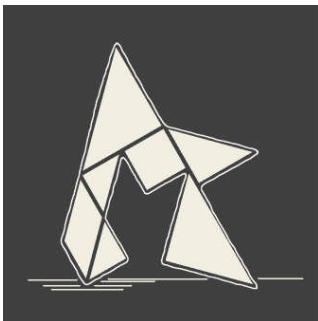


N. 6, 7 ►

N. 8, 9 ►



◀ N. 4



◀ N. 5

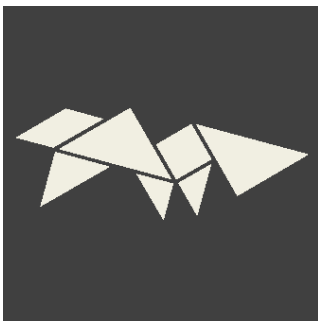
N. 10-12 ▶



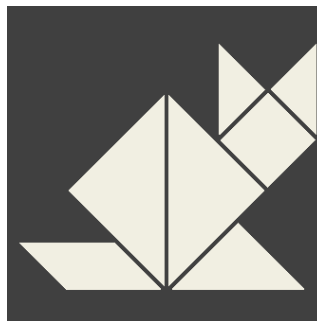
N. 13-15 ▶



N. 16-18 ▶



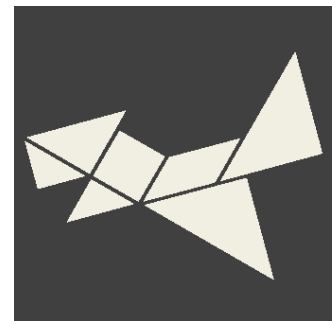
▲ N. 19



▲ N. 20

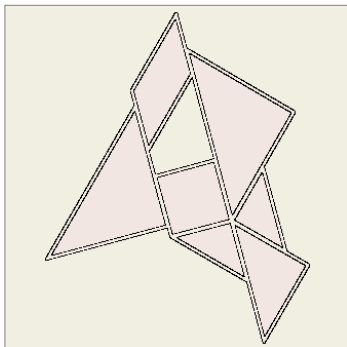


▲ N. 21

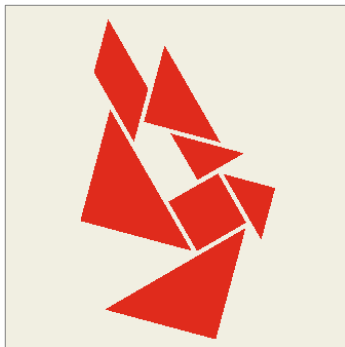


▲ N. 22

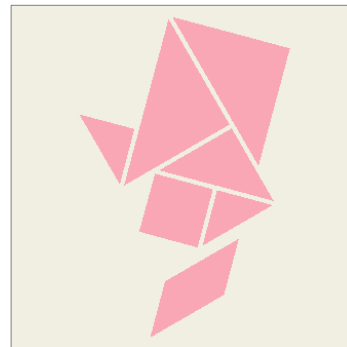
Faces of all colours



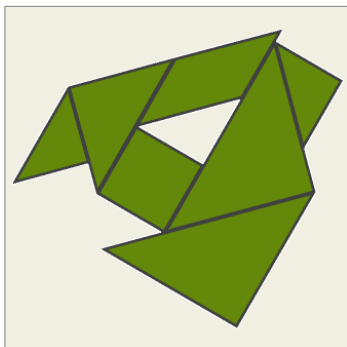
▲ N. 23



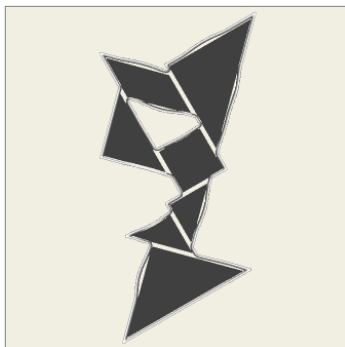
▲ N. 24



▲ N. 25



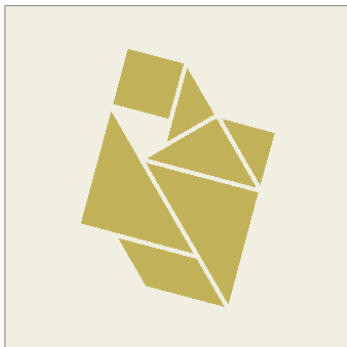
▲ N. 26



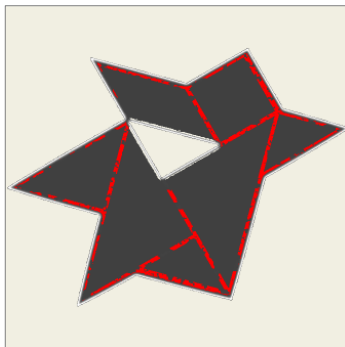
▲ N. 27



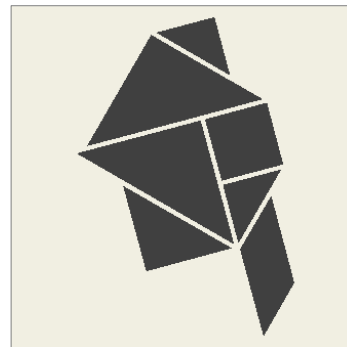
▲ N. 28



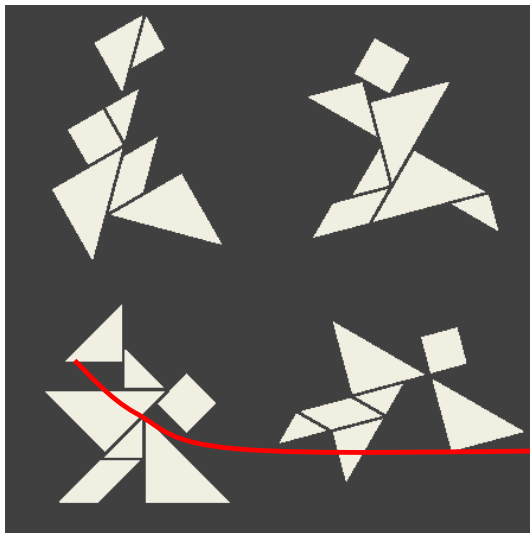
▲ N. 29



▲ N. 30



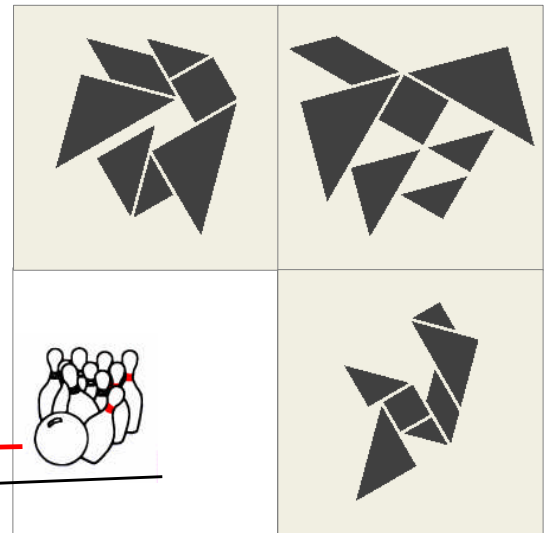
▲ N. 31



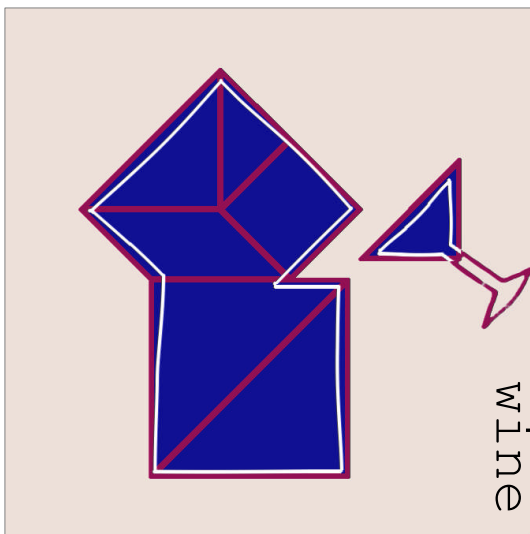
◀ N. 32, 33

N. 36, 37 ▶

◀ N. 34, 35



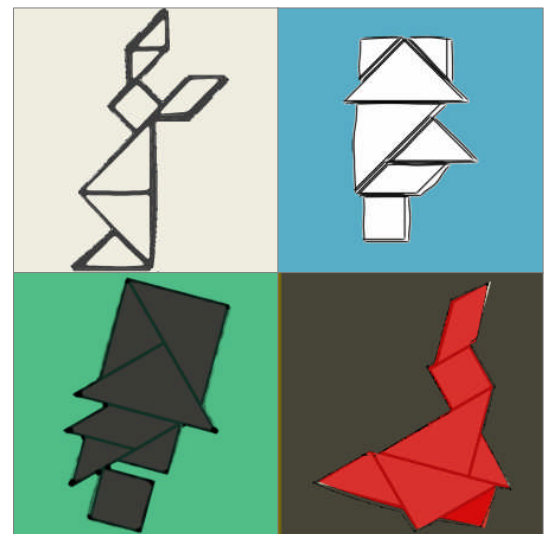
▲ N. 39

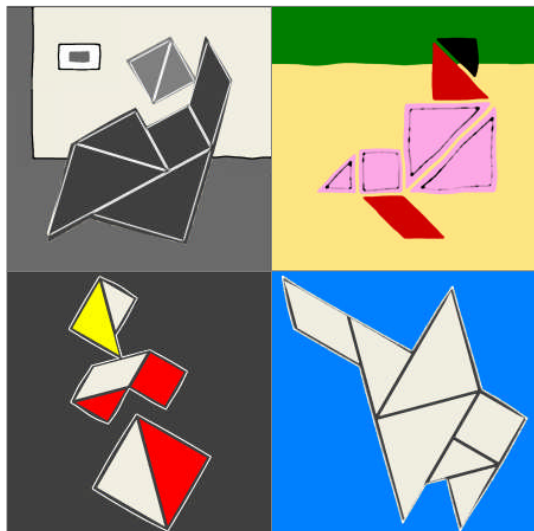


N. 40, 41 ▶

◀ N. 38

N. 42, 43 ▶



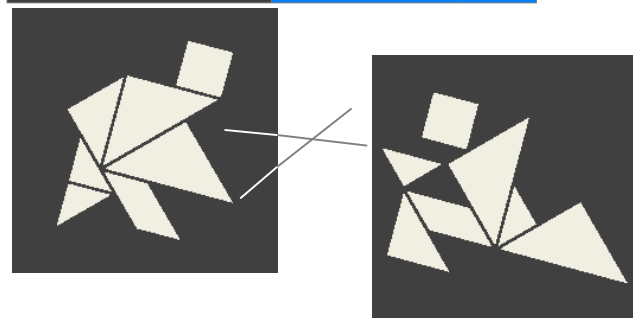


◀N. 44, 45

N. 48, 49▶

◀N. 46, 47

N. 50, 51▶



◀N. 52, 53

N. 56, 57▶



◀N. 54, 55

N. 58, 59▶

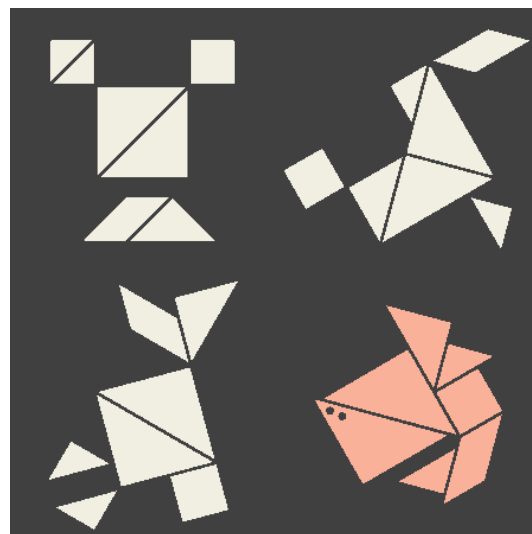
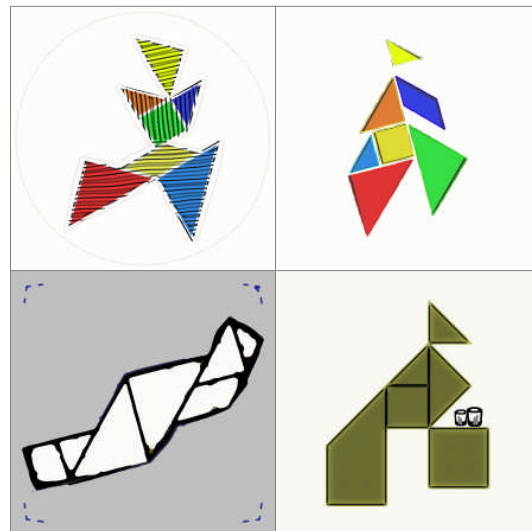
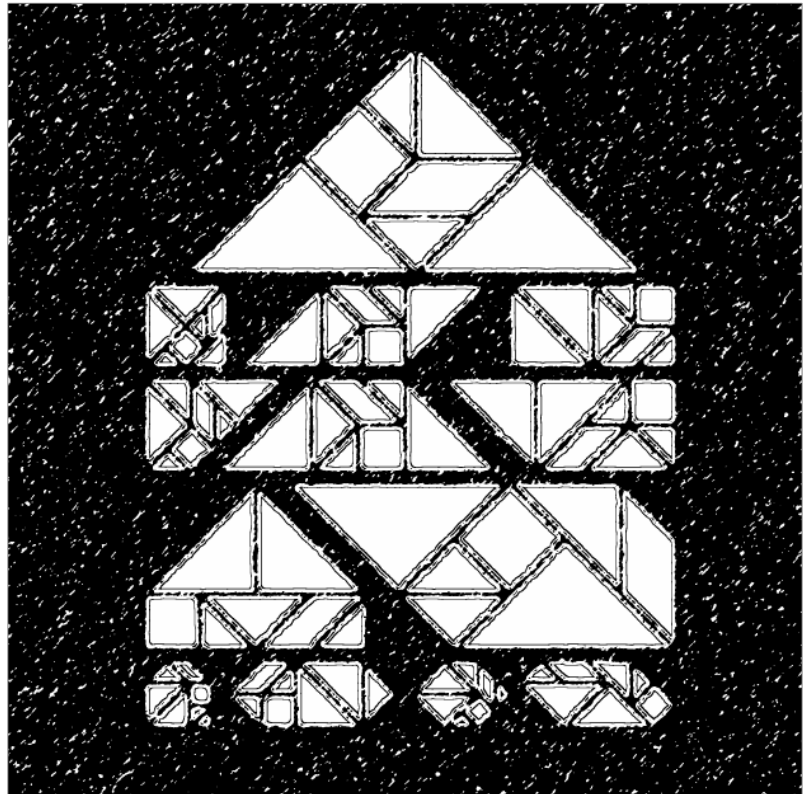


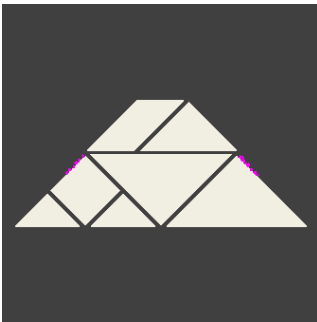


Figure 2 ►  
*Inside the thirteen's home*  
 2500x2500 giclée  
 reworked from an original picture from  
 geocities/tangmath

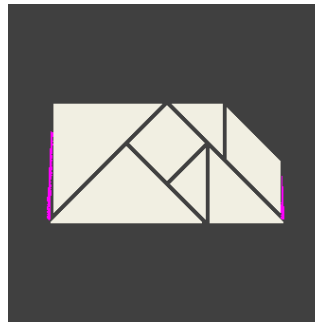


Nearly a trapeze: The trick is based on the small difference between  $2\sqrt{2} \approx 2.83$  (the hypotenuse of the large triangle) and 3 (the sum of the sides of the rhomboid and the medium triangle).

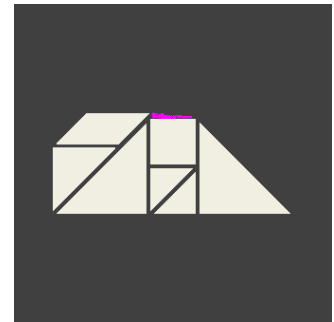
▼ N. 68



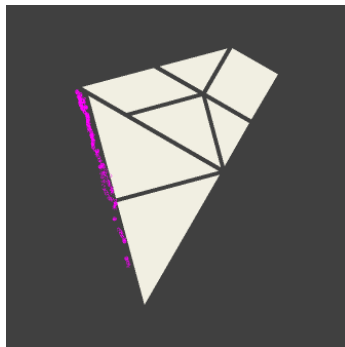
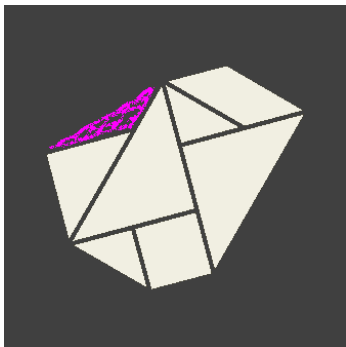
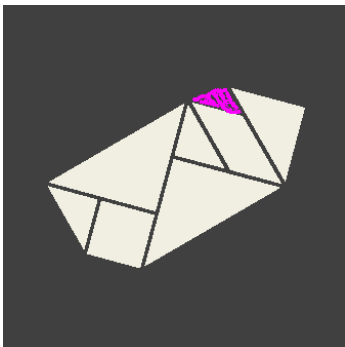
N. 70 ►  
 Nearly a pentagon



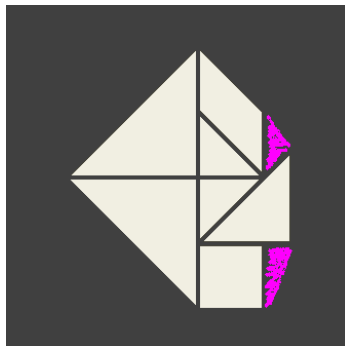
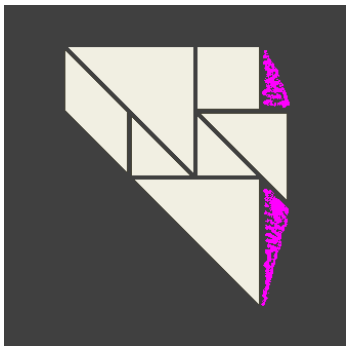
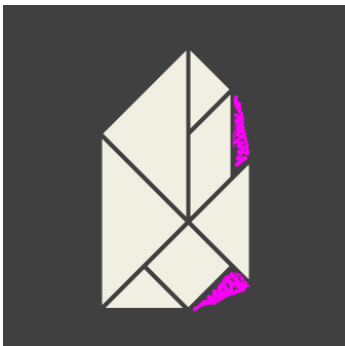
N. 73 ►  
 Nearly a pentagon



Some Geometry - Solutions

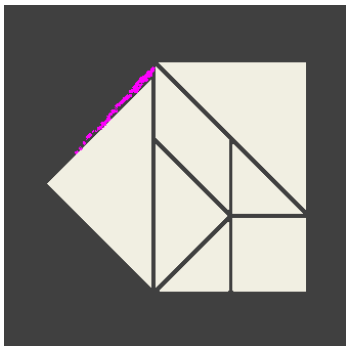
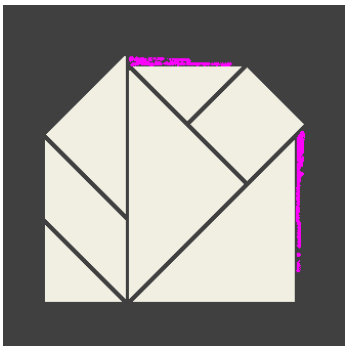


◀N. 78-80  
from left to right

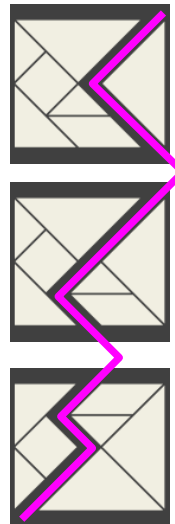


◀N. 81-83  
from left to right

N. 86-88  
from top to bottom ▼

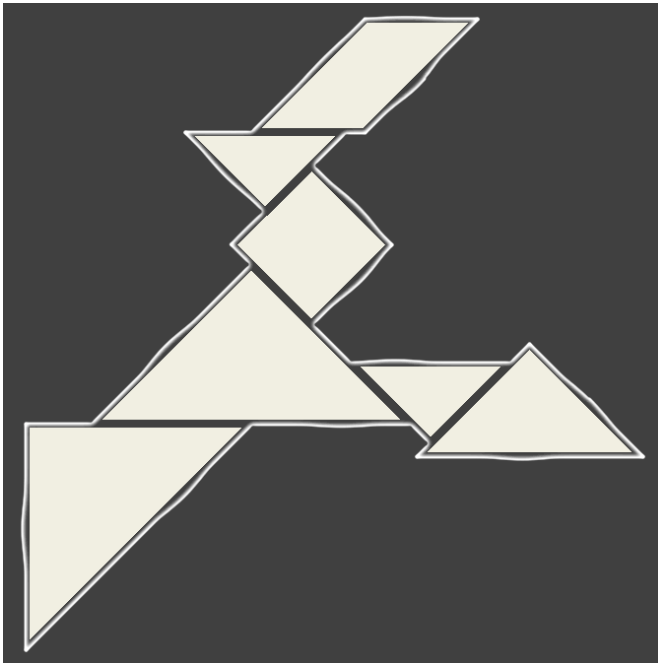


◀N. 84, 85  
from left to right



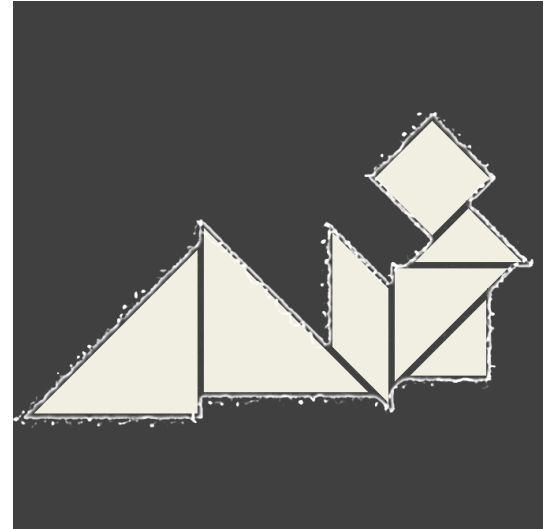
N. 89►

No matched vertices

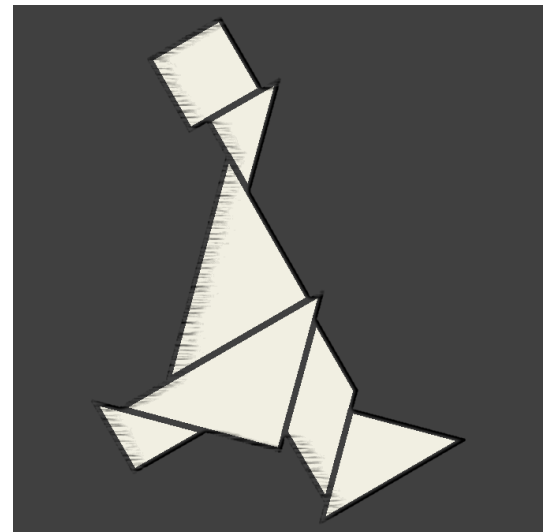


▲ N. 90

*Unique Forms of Continuity in Space* (1913) by Umberto Boccioni



N. 91►





◀ N. 92, 93



▲ N. 94



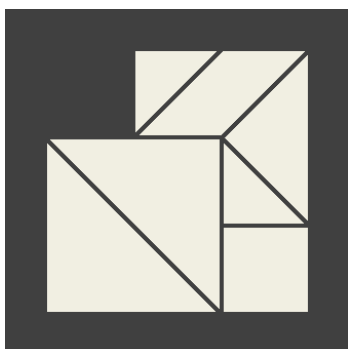
▲ N. 95



▲ N. 96



▲ N. 97



▲ N. 99



▲ N. 100



▲ N. 101



◀N. 102  
A parallelogram from a second tangram set is used instead of the medium triangle.

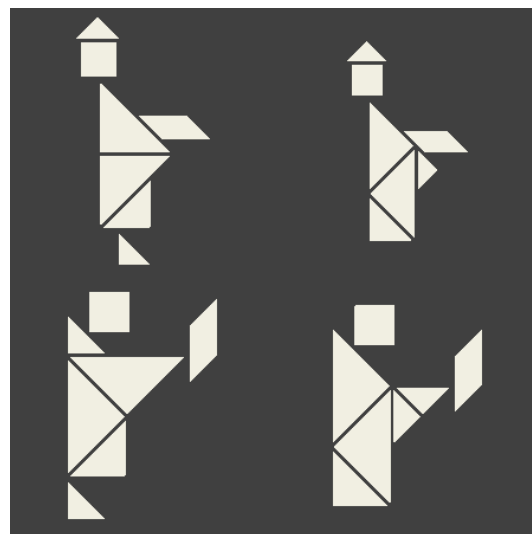
N. 104 ▶  
My solution to Loyd's paradox. The trick is based on the small difference between  $2\sqrt{2} \approx 2.83$  (the hypotenuse of the large triangle) and 3 (the sum of the sides of the rhomboid and the medium triangle).

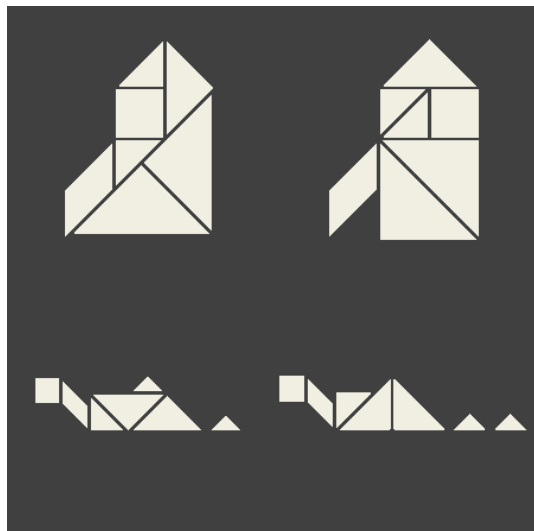


◀N. 103  
A medium triangle from a second tangram set is used instead of the parallelogram.

N. 105, 106 ▶

N. 107, 108 ▶





◀ N. 109, 110

N. 117 ▶

A small triangle from a second tangram set is used instead of the parallelogram.

◀ N. 111, 112

N. 118 ▶

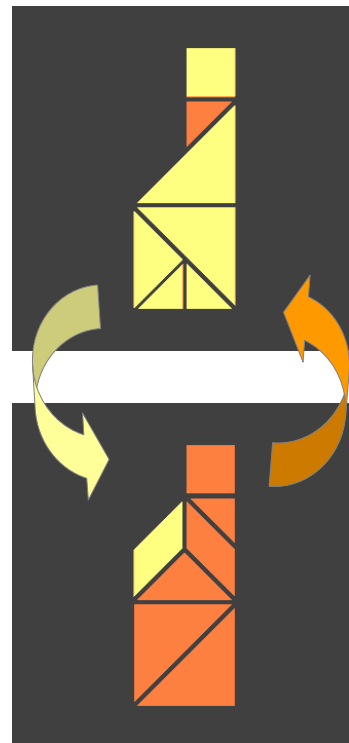
A parallelogram from a second tangram set is used instead of a small triangle.

◀ N. 113, 114

N. 119 ▶

The cabinet organ with the folding lid is a true tangram.

◀ N. 115, 116





◀N. 121-124

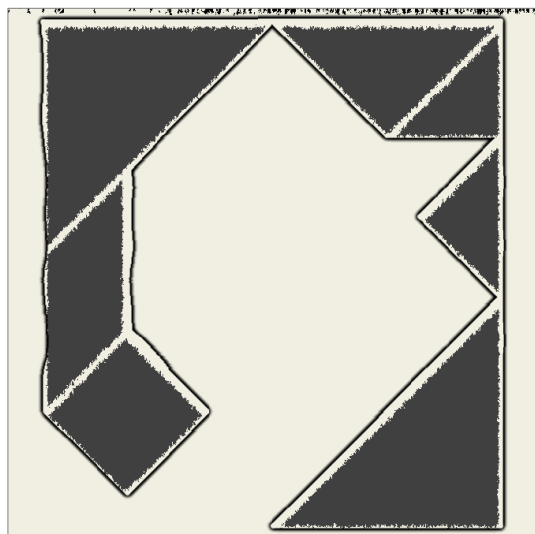
N. 125128▶



◀129-131



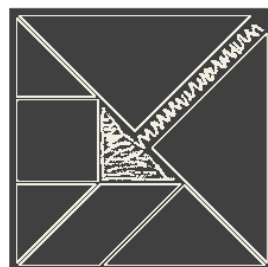
◀132-134



◀ N. 121

▶ N. 120

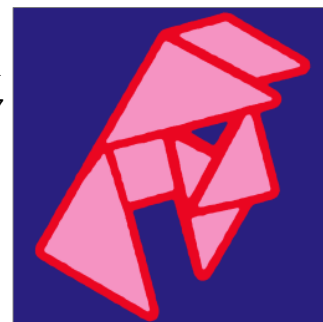
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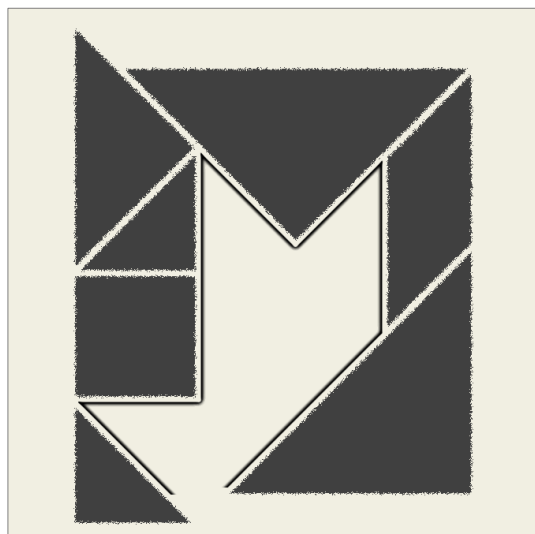
▶ N. 160



▶ N. 177



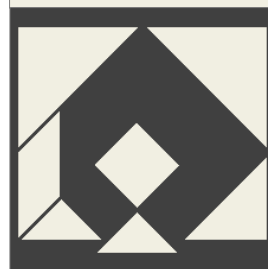
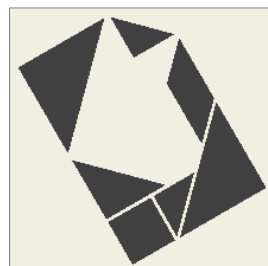
▶ N. 194



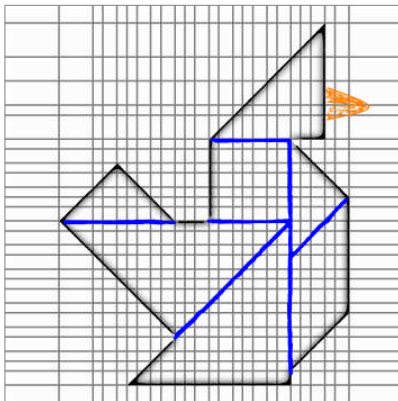
◀ N. 122

▶ N. 124

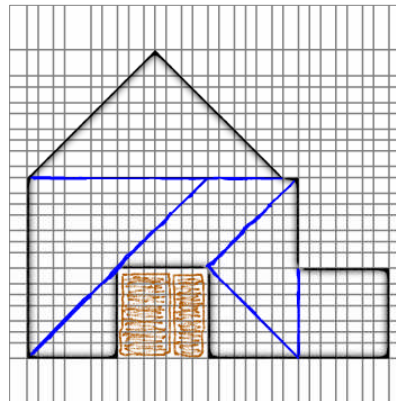
▶ N. 125



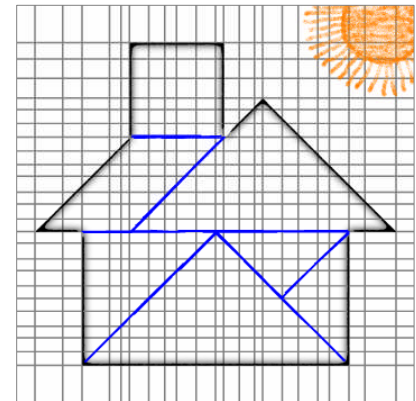




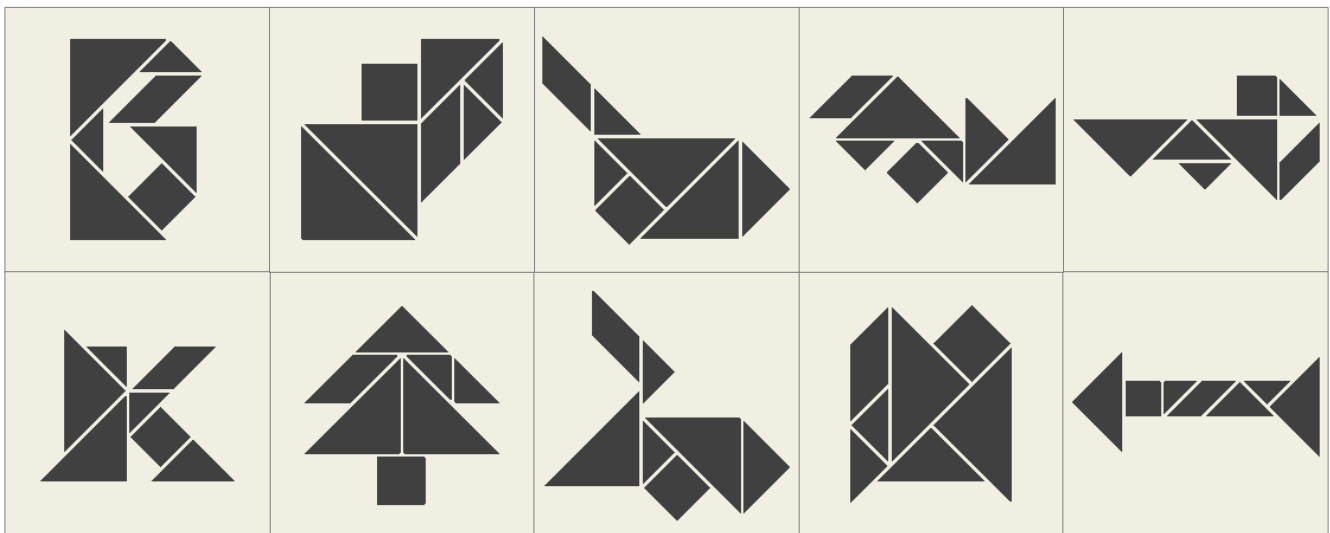
▲ N. 211



▲ N. 214



▲ N. 215



▲ N. 212, 213

▲ N. 216, 217

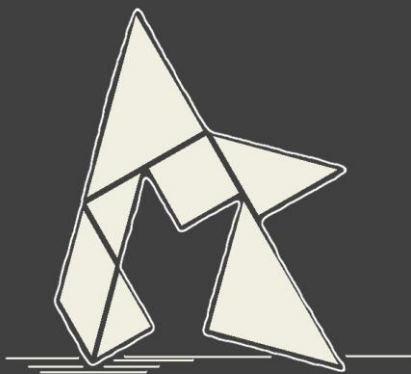
▲ N. 218, 219

▲ N. 220, 221

▲ N. 222, 223







## Wandering around Tangram

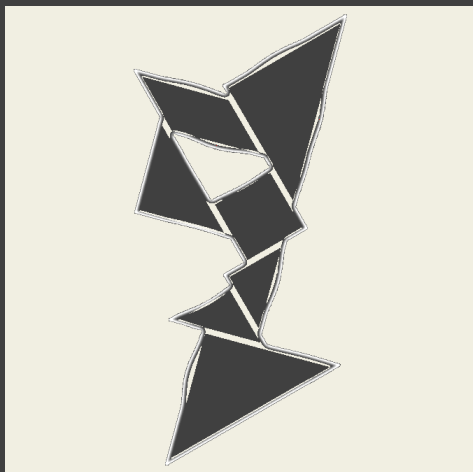
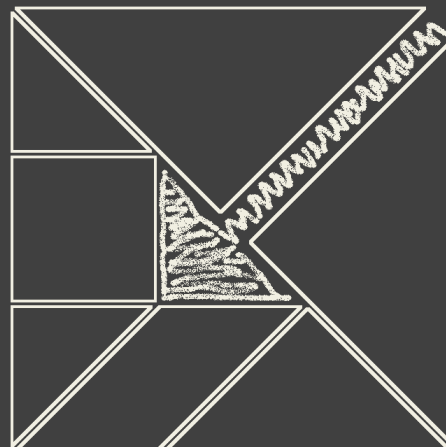
Franco Cocchini

*Exploring the fascinating world of Tangram*

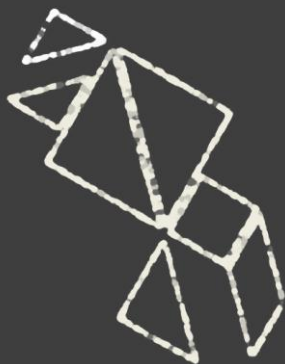
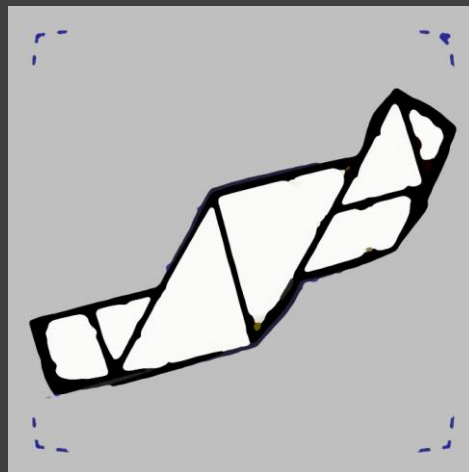
*For all the ages*

*Tangram pictures as you ever seen, still preserving the game side*

*More than 200 shapes, with full solutions*



**Franco Cocchini** is the author of several patents and scientific publications on Solid State Physics, Rheology and Lightwave Technologies. He shares his interest on puzzles in the site [www.tanzzle.com](http://www.tanzzle.com)



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